

# VOCAL TRACT ACOUSTICS USING THE TRANSMISSION LINE MATRIX (TLM) METHOD

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## ABSTRACT

Most traditional theories of speech production are currently based on plane waves and on one-dimensional analysis. It is however well-known that when the frequency of sound reaches a cut-on frequency, higher acoustical modes start to propagate and can become predominant. It is therefore important to evaluate the effects of these higher modes, especially in order to improve acoustical models of the vocal tract. This paper describes a new numerical method to study the propagation and the radiation of speech sounds, and to compute acoustic characteristics of the vocal tract. This method, named Transmission Line Matrix or Modelling (TLM), has been used for simulating electromagnetic wave propagation and is used here for the first time in acoustics. The TLM method provides time domain solutions in 2D and 3D spaces. The main advantage of this method is the simplicity of formulation and programming for a large range of applications. We first describe the principle on which the TLM method is based. The method as well as the boundary conditions used are validated using classical tests. A systematic study of higher order mode propagation and radiation is then presented. We focus on the influence of some critical parameters such as vocal tract width and location of the sound source. In particular, we show how, using TLM simulation, it is possible to derive modal reflection and transmission characteristics of the vocal tract. A typical example of simulation is presented and discussed.

## 1. PRINCIPLE OF TLM METHOD

Transmission Line Matrix (TLM) or Transmission Line Modelling is a general numerical method suitable for simulating three dimensional electromagnetic fields in complex geometries. A first application to acoustic wave propagation problems has been proposed and validated by El-Masri *et al.* (1996).

The principles of the TLM time domain method have been introduced by Johns & Beurlé (1971). Waves are represented by a discrete spatial electrical network model (Transmission line matrix). Voltages and currents in this network are equivalent to electric and magnetic fields in electromagnetic systems. Propagation of electromagnetic fields is simulated by the propagation and scattering of pulses in a network consisting of interconnected ideal transmission lines. At each

time step, every node receives incident voltage pulses, and sends scattered pulses, as shown in Figure 1. A scattering matrix determines the relationship between incident and scattered pulses. The scattered pulses at time  $(t)$  become incident pulses on adjacent nodes at  $(t + \Delta t)$ . The scattering matrix is computed from transmission lines theory.

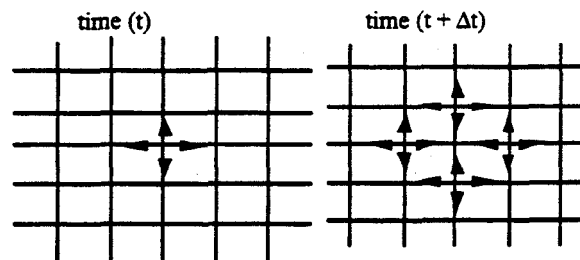


Figure 1: Propagation of impulsions in a two dimensional network (after Saguet, 1985).

## 1.2. Equations of TLM in 3 Dimensional of Space in Acoustics

Each node from the network represents a parallel junction of three transmissions lines. The equivalent electric scheme of a scalar basic node is shown in Figure 2.

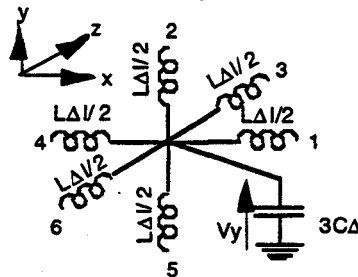


Figure 2: Electric scheme of a scalar node in 3 dimensions

$L$  and  $C$  are the inductances and capacitances per unit length for the transmissions lines. The distance between two adjacent nodes is constant, equal to  $\Delta l$ . The relationship between voltage and currents for this junction is represented by the following differential equation:

$$\vec{\nabla} V_y = -L \frac{\partial \vec{I}}{\partial t} \quad (1)$$

where  $V_y$  is the voltage on the node, and  $\vec{I} (I_x, I_y, I_z)$  is the current. The current conservation on the node gives the equation:

$$\frac{\partial I_x}{\partial x} + \frac{\partial I_y}{\partial y} + \frac{\partial I_z}{\partial z} = -3C \frac{\partial V_y}{\partial t} \quad (2)$$

From equations (1) and (2), the wave equation becomes:

$$\Delta V_y = 3LC \frac{\partial^2 V_y}{\partial t^2} \quad (3)$$

For the sound pressure  $P$ , the acoustic wave equation is:

$$\Delta P = \frac{1}{c^2} \frac{\partial^2 P}{\partial t^2} \quad (4)$$

where  $c$  is the speed of sound.

It is clear that there is a direct analogy between equations (3) and (4). In that way we can find an analytical equivalence between voltage  $V_y$  and sound pressure  $P$ , and between current  $\vec{I} (I_x, I_y, I_z)$  and sound velocity  $\vec{v} (v_x, v_y, v_z)$ .

### 1.3. Boundary Conditions

In order to ensure that the scattered pulses come back in synchrony with initial incident pulses at a given node, the boundaries are placed at equal distances from adjacent nodes. For a rigid wall, the normal velocity on the wall is equal to zero, i.e. the reflection coefficient equals 1. Free field boundary conditions can be simulated by using Taylor's series expansion (Saguet, 1991):

$$V_{yr}(N_y, t) = 2.5V_{yr}(N_y - 1, t - 1) - 2V_{yr}(N_y - 2, t - 2) + 0.5V_{yr}(N_y - 3, t - 3) \quad (5)$$

where  $V_{yr}(N_y, t)$  is the reflection coefficient at node  $N_y$  and at time  $t$  along the  $y$ -axis. This simple procedure is depicted in Figure 3.

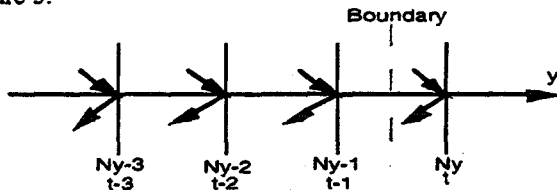


Figure 3: Simulation of infinite space boundary conditions.

## 2. VALIDATION OF TLM METHOD

A systematic validation of the TLM method has been performed, focusing in particular upon the description of sound sources and on boundary conditions. Examples of such tests (monopole, quadrupole sound sources, description of free field conditions) can be found in El Masri *et al.* (1996). As another interesting test, we computed the transfer function for a Helmholtz resonator. The dimensions of the back cavity were (3,9 cm, 6 cm, 3.3 cm) and those of the neck (1.5 cm, 3 cm, 0.9 cm). Figure 4 presents the results of TLM simulations and the theoretical prediction.

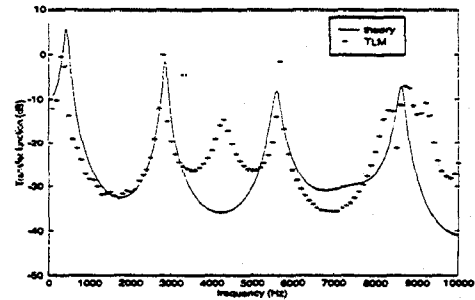


Figure 4: Transfer functions for a Helmholtz resonator obtained by theory and TLM.

We observe a reasonable agreement between theory and TLM simulations, except at 4300 Hz and above 6000 Hz. These discrepancies are due to the apparition of propagating transverse modes in the back cavity. A more detailed study of these modes is presented in the following.

## 3. ACOUSTICS OF VOCAL TRACT AT HIGH FREQUENCIES - EFFECT OF HIGHER ORDER MODES

### 3.1. Theory

A general solution for the propagation of sound waves in a pipe can be expressed in the frequency domain as:

$$P = \sum_{m,n} A_{mn} \cdot \frac{\Psi_{mn}(R) \cdot \Psi_{mn}^*(S)}{k_{mn}^2 - k^2} \cdot (e^{-j \cdot k_{mn} \cdot y} + R_{mn} e^{j \cdot k_{mn} \cdot y}) \quad (6)$$

where  $A_{mn}$  is constant,  $\Psi_{mn}$  and  $k_{mn}$  are the eigen function and eigenvalues of the associated Helmholtz problem,  $y$  is the longitudinal axis,  $S$  is the position of the sound source,  $R$  the position of the receiver and  $R_{mn}$  the reflection coefficient at the end of the pipe.

In the case of a rigid rectangular geometry:

$$\Psi_{mn}(x, z) = \cos\left(\frac{m\pi}{Lx} x\right) \cdot \cos\left(\frac{n\pi}{Lz} z\right) \quad (7)$$

where  $L_x$  and  $L_z$  are the transverse direction of the duct. The associated acoustical wave number is:

$$k_{mn}^2 = \frac{\omega^2}{c^2} - \frac{m^2\pi^2}{L_x^2} - \frac{n^2\pi^2}{L_z^2} \quad (8)$$

From equation (8), we see that for one frequency excitation below the cut-on frequency  $f_{mn}$  given by:

$$f_{mn} = \frac{c}{2} \sqrt{\frac{m^2}{L_x^2} + \frac{n^2}{L_z^2}} \quad (9)$$

$k_{mn}$  is purely imaginary and the mode  $(m, n)$  will be evanescent. Above this cut-on frequency the mode  $(m, n)$  will be propagating. From physiological considerations, Pelorson *et al.* (1995) estimated the first cut-on frequency to lie within the range 5-10 kHz. This has been confirmed by experimental or numerical studies (Lu *et al.*, 1993, Motoki *et al.*, 1992). An accurate description of higher modes propagation is found to be particularly crucial when considering the radiation of speech at the lips (Pelorson *et al.*, 1995). The classical plane piston model used in speech literature (Flanagan, 1972) is then clearly inaccurate.

### 3.2. A Study of Higher Modes by TLM Simulation

In this section, we consider the termination of the vocal tract (i.e. the lips) as a rectangular uniform section which is a simplified, but not unrealistic approximation of the actual elliptical shape. We define the duct aspect ratio ( $\beta=L_z/L_x$ ) as the ratio between the height and the width of the duct.

**Location of the sound source.** Equation (7) clearly indicates the crucial role of the position of the sound source for the generation of higher modes. In particular, for a source located on a transverse boundary ( $x_s=L_x, z_s=L_z$ ), the efficiency of higher modes is predicted to be maximum. Conversely, when the sound source is located on the axis of the duct ( $x_s=L_x/2, z_s=L_z/2$ ), the higher modes will not be generated. To illustrate this, we present a comparison between theory and TLM simulation for the amplitude of the first transverse mode as a function of the position of the sound source (see figure 5). The agreement between theory and simulation is excellent.

**Reflection coefficient of higher modes.** In order to analyse and to measure the effects of higher modes on radiation characteristics, two simulations were performed. In the first one, the sound source (monopolar) was located on the axis of the duct ( $x_s=L_x/2, z_s=L_z/2$ ) according to equation (7) and to the section above, the first transverse mode will not be generated. In a second simulation, the sound source was located in a corner of the duct. In this case the effects of the transverse mode are maximum. By a simple subtraction of pressure and velocities

simulated in both cases, we are therefore able to isolate the transverse mode and to measure its radiation characteristics. Figure (6) presents examples of results for aspect ratios  $\beta$  typical of speech (Pelorson *et al.*, 1995). These simulations show the dramatic influence of higher modes upon the radiation characteristics. Note that the associated cut-on frequencies  $f_c$  lie within the limit  $3.2 \text{ kHz} < f_c < 5.5 \text{ kHz}$ .

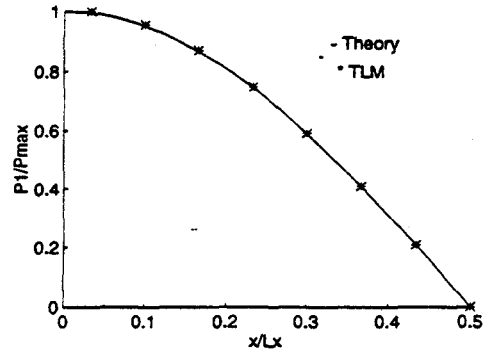


Figure 5: Effect of the source position on the amplitude of the higher order mode.  $x/L_x = 0$  corresponds to a corner excitation and  $x/L_x = 0.5$  corresponds to a centred excitation.

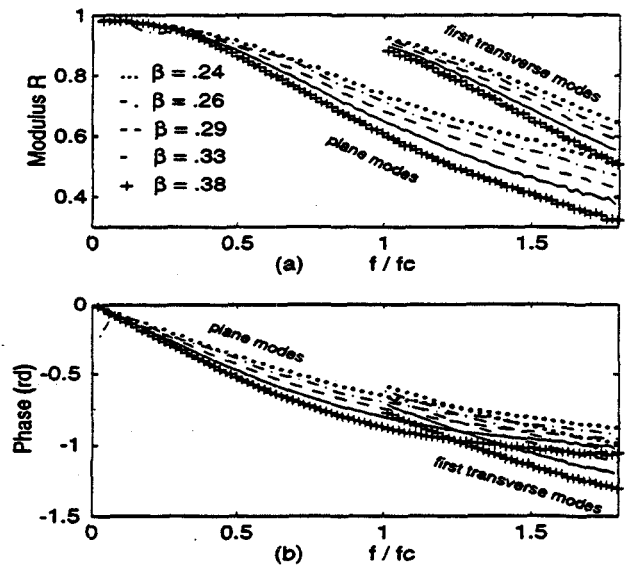


Figure 6: Modulus and phase of the reflection coefficient of the plane and first transverse mode as a function of the dimensionless frequency  $f/f_c$ , where  $f_c$  is the cut-on frequency of the first transverse mode. The parameter is the aspect ratio  $\beta$ .

### 3.3. Derivation of a Model

As a theoretical account for higher modes still remains difficult and time consuming, TLM simulation can provide an empirical basis for an improved description of the vocal tract acoustics at high frequencies. To illustrate this, we present in figure (7) a

comparison between the measured radiated sound pressure field at the end of a rectangular pipe ( $L_x=4\text{cm}$ ,  $L_y=60\text{cm}$ ,  $L_z=1.5\text{cm}$ ) and the corresponding TLM simulation at 4.5 kHz. Details of the measuring technique can be found in (Pelorson *et al.*, 1995). In this figure are also presented the theoretical expectation using the classical plane piston model and a new expectation using the reflection coefficient of the first transverse mode predicted by the TLM method.

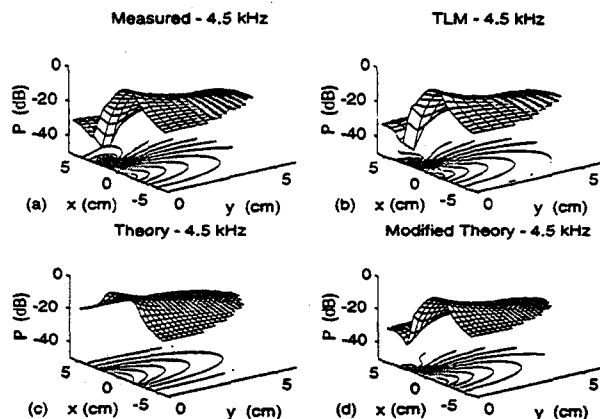


Figure 7: (a) Measured and (b) simulated radiated sound pressure field. (c) Plane piston theory predictions, (d) theory accounting for the first transverse mode.

This example clearly shows the improvement gained by the new theory in particular concerning the directive effects of the first higher mode.

### 3.4. Vocal Tract Simulations

As a last example, we present results of simulations obtained for a vocal tract geometry corresponding to a vowel /a/. Although effects of higher modes are not expected to be spectacular in such a case, due to the centred position of the sound source in phonation. This study allows a comparison with results obtained by Finite Element Methods (Lu *et al.*, 1993). The results obtained at 5.2 kHz (see Figure 8) are consistent with the observations of Lu *et al.* (1993). As expected, we observe the apparition of transverse modes in the widest sections of the vocal tract.

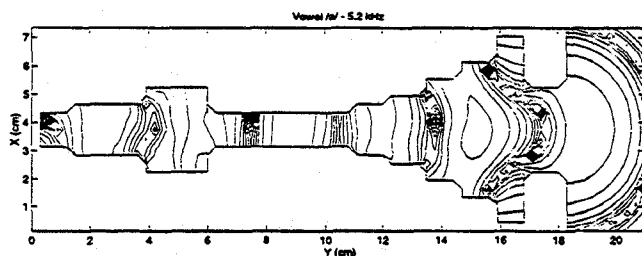


Figure 8: Sound pressure contours for vowel /a/ – 5.2 kHz.

## 4. CONCLUSION

In this paper we have described an original numerical method to simulate the acoustics of the vocal tract. Compared with traditional FEM methods, this technique has the main advantage to perform time domain simulations in a very simple and accurate way. A study of higher acoustical modes has been presented. We showed how, using the TLM method, it was possible not only to evidence these modes but also to measure their characteristics. Based on these findings, a new propagation model has been derived and appears to be much more accurate than the classical plane wave model. First simulations using complex vocal tract geometries tend to confirm the importance of these higher modes.

## 5. ACKNOWLEDGEMENTS

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