The stiffness of a consolidated granular medium experiences a drop immediately after a moderate mechanical solicitation. Then the stiffness rises back toward its initial value following a logarithmic time evolution called slow dynamics. In the literature, slow dynamics has been probed by macroscopic quantities averaged over the sample volume, for instance by the resonant frequency of vibrational eigenmodes. This article presents a different approach based on Diffuse Acoustic Wave Spectroscopy (DAWS), a technique that is directly sensitive to the details of the sample structure. The parameters of the dynamics are found to depend on the damage of the medium. Results confirm that slow dynamics is, at least in part, due to tiny structural rearrangements at the microscopic scale, such as inter-grain contacts.

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I. INTRODUCTION: WHAT IS SLOW DYNAMICS?

Depending on the magnitude of the solicitation applied to it, a consolidated granular medium, like sedimentary rocks or concrete, can asymptotically react either in the elastic (reversible) regime, or in the brittle regime where irreversible fractures develop. A few years ago, several articles from Guyer, TenCate and co-workers [TenCate and Shankland (1996); Guyer et al. (1998); Guyer and Johnson (1999); TenCate et al. (2000)] reported on an intermediate regime where they observed non-linear elasticity. This regime is observed after imposing a strain of moderate amplitude \(10^{-6}\) that does not generate any macroscopic damage. They called its time-evolution slow dynamics: it starts with a drop of the elastic modulus, followed by a logarithmic recovery of the sample stiffness toward its initial value after the strain is released. The \(\log(t)\) recovery was found to be universal in granular solids of various composition [TenCate et al. (2000); Lacouture et al. (2003); Johnson and Sutin (2005)], humidity [Vakhnenko et al. (2004)] or level of damage [Van Den Abeele et al. (2001)]. Most of the time, the evolution of the sample stiffness is monitored through the frequency of vibrational eigenmodes, a technique named Non-Linear Elastic Wave Spectroscopy [Guyer and Johnson (1999)]. More recently, Lobkis and Weaver (2009) proposed to monitor the stiffness of the material using the Larsen effect between two piezo-electric transducers. The resonant frequency \(f_L\) observed with a feedback loop was measured with a relative precision of the order \(\frac{\delta f_L}{f_L} \approx 10^{-6}\), and was found to satisfactorily monitor the rigidity of the material. This latter technique offers simultaneously an increased sensitivity and a temporal resolution of a few ms, thus allowing to study slow dynamics over very short time-scales.

In this article, we develop an alternative method based on broadband diffuse ultrasound. We take advantage of coda waves that can be observed in multicomposite materials where multiple scattering effects (characterized by the scattering mean free time \(t^\ast\)) dominate absorption effects (characteristic time \(\tau_{abs}\)). Denoting by \(T\) the central period of the ultrasound, and by \(\Delta d\) the time between two ultrasonic records, the time scales in our experiment follow the hierarchy:

\[ T < t^\ast < \tau_{abs} < \Delta d. \]  

Similarly to Coda Wave Interferometry (CWI) [Poupinet et al. (1984); Roberts et al. (1992); Snieder et al. (2002)] and to Diffuse Acoustic Wave Spectroscopy

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(DAWS) [Pine et al. (1988); Cowan et al. (2002)], we study weak changes of the diffuse waveforms in amplitude and phase. We compare waveforms recorded after a moderate solicitation to those recorded before it. The solicitation consists in the impact of a small steel ball falling onto the sample from a controlled height (one meter in general). The composition of the samples is displayed on table I.

<table>
<thead>
<tr>
<th>Composition (in weight)</th>
<th>Weight density</th>
</tr>
</thead>
<tbody>
<tr>
<td>cement: 17%</td>
<td>12.5 kg</td>
</tr>
<tr>
<td>fine sand: 31%</td>
<td>2.2 kg/l</td>
</tr>
<tr>
<td>gravel: 43%</td>
<td></td>
</tr>
<tr>
<td>water: 9%</td>
<td></td>
</tr>
</tbody>
</table>

TABLE I. Sample composition

II. EXPERIMENTAL SETUP

In the first experiment, we use a cylindrical sample of 16 cm in diameter and 30 cm in height (sample #1). Two ultrasonic transducers are used: one as a source and one as a receiver (labeled S and R, respectively, in Fig. 1); they are 10 cm apart. Note that the lateral part of the transducer is inactive and rigid, so that surface (Rayleigh) waves are neither excited nor recorded. In this sample, the scattering mean free time is of the order of 12µs, the transit time is of the order of 60µs, and the absorption time about 200µs.

The lateral size of the active part of the transducers (0.7 mm) is much smaller than the wavelength (\(\lambda = 5 \text{ mm to 5 cm at working frequencies}\)), which makes them very sensitive to multiply scattered waves. The transducers are glued onto the sample using a hot chemical glue (phenyl-salicylic acid) that solidifies with cooling (below 43°C). The ultrasonic experiment was performed several months after casting the samples. We send a 80V broad-band ultrasonic impulse with frequencies ranging from 50 to 500 kHz. The signal measured by the receiver is amplified, digitized and stored in memory. A typical record \(h_d(t)\) is shown in Fig. 2. The impulse response contains a hardly visible direct arrival followed by a long lasting coda due to multiply scattered waves. The acquisition is repeated at various dates \(d\) (\(d = 0\) starts the experiment), over about one hour. At date \(d_f\), we drop a small metallic ball (\(m = 30 \text{ g}\)) from a height \(h = 1.38 \text{ m}\) onto the top face of the sample. The impact is about 15 cm from the source and the receiver. The elastic energy released to the sample is: \(E_{kin} = mgh \approx 0.4 \text{ J}\). Note that we observe no visible damage caused by this moderate impact.

![Typical record collected through a concrete sample. The integration time-window is marked by the two dashed vertical lines: at 0.05 ms and at 0.65 ms. The initial impulse duration is about 10µs.](image)

III. DATA PROCESSING

From the set of ultrasonic data obtained during an experiment, we derive two quantities that depend on \(d\). First, we measure the relative velocity change \(\delta V_d/V\) within the material [Poupinet et al. (1984); Snieder et al. (2002); Lobkis and Weaver (2003)]. Second, we measure the remnant decorrelation \(K_d\) of the waveforms after correcting for the effect of the relative velocity change [Lobkis and Weaver (2003)].

A. Relative velocity change \(\delta V/V\) (CWI)

Let us describe the two steps of our data processing in details. To measure the relative velocity change, we compare the phase of the waveforms acquired at the date \(d\) to the phase of the initial record \((d = 0)\). Following Sens-Schönfelder and Wegler (2006), we interpolate and resample \(h_d(t)\) at times \([1 + \epsilon]t\); this corresponds to stretching the time-axis of the record by a factor \(1 + \epsilon\).

Then we evaluate the correlation coefficient between \(h_0\) and \(h_d\):

\[
X_d(\epsilon) = \frac{\int h_0(t)h_d([1 + \epsilon]t)dt}{\sqrt{\int h_0^2(t)dt} \sqrt{\int h_d^2([1 + \epsilon]t)dt}},
\]

where the integration is performed over a time-window larger than the period \(T\). This time-window is marked by two vertical broken lines in fig. 2. This calculation is repeated for various values of the parameter \(\epsilon\). The parameter \(\epsilon\) maximizing \(X_d\) corresponds to the actual relative velocity change:

\[
\epsilon_{max} = -\delta V_d/V.
\]

The evolution of \(\delta V_d/V\) as a function of \(d\) is displayed in fig. 3 (top). Note that the ball impact occurs at date \(d_f = 270 \text{ s}\) after starting the experiment. Before the impact, there is no relative velocity change, indicating that the medium is at rest. Immediately after the impact, the velocity is noticeably decreased. Since the density of the material is not changed, and since diffuse waves are mainly in the transverse (shear) polarization,

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The decorrelation waveform decreases over several hundreds of seconds. Reference, then the decorrelation from the reference ultrasonic waveform differs significantly from the one released by the ball. Here, it corresponds to a feeble change in the scattering properties of the concrete sample [Lobkis and Weaver (2003); Snieder et al. (2002); Larose et al. (2006)], and consequently the decorrelation. Relative velocity changes due to temperature variations are found to be negligible in our experiment, as can be noticed by the flat curve in fig. 3, from t=0 to t=270 s. The decorrelation of the waveforms before the impact is actually near 0.1 %, and is attributed to the noise generated by the electronic devices. We neglect it in the following. Note also that the changes of intensity of the diffuse waveforms (say, the quality factor of the coda) was observed to show some recovery [TenCate et al. (2000)], but this effect is not studied here and is beyond the scope of the present paper.

IV. THE PHYSICAL PARAMETERS OF SLOW DYNAMICS

The time evolutions of both $\delta V/V$ and $K$ fit logarithmic laws (with $d$ and $d_I$ in seconds):

$$\delta V/V = A' - m' \log_{10}(d - d_I),$$
$$K_d = A - m \log_{10}(d - d_I).$$

Such a logarithmic evolution, valid if $d \ll 10^4/m$ is referred to as slow dynamics. $A$ and $m$ are the fit parameters: $A$ is the extrapolated decorrelation 1 s after the impact, and $m$ the slope of the logarithmic decay. $10^4/m$ represents the recovery time (in seconds): the time the system needs to recover its initial state. In general, this recovery time is of the order of $10^4$ to $10^5$ s. Parameters $A$ and $m$ were found to depend on the experimental apparatus, but Eq. (6) was found to describe the dynamics in all cases. TenCate et al. (2000) mentioned a break away from the log recovery after 1000 s. In our experiments, we should certainly expect an erosion of the log dynamics at some time. In particular, we must consider the possibility that part of the change in our experiments might be irreversible, and that the recovery might saturate. This could be ascribed to irreversible (but invisible) damages caused by the impact. As we could not monitor the sample longer than 1000s, we can not address this issue properly.

A. Slow Dynamics versus time in the coda

Let us now study the parameters $A$ and $m$ in more details. As a first test, we evaluate these parameters at different times $t$ in the coda. To that end, we split the initial time window (see fig. 2) into five consecutive time windows of length 0.12 ms. The decorrelation $K_d$ for these five windows is displayed in Fig. 4. $A$ and $m$ were found to increase with the time in the coda, confirming that late diffuse waves are much more sensitive to weak perturbation than early arrivals. Interestingly enough, the recovery time $10^4/m$ is constant within the whole record, which confirms that all time-windows (thus all wave paths) undergo the same slow dynamics.
FIG. 4. Decorrelation versus the date of the measurement (in log scale) for various time windows in the coda. The central time of each window is displayed in the box. The recovery time $10^{\frac{A}{m}}$ is found to be constant in the whole record.

phenomenon.

B. Slow Dynamics versus frequency

To study the frequency dependence of slow dynamics, we filter the whole record into different frequency bands: we observe that $A/m$ is constant in the 50-500 kHz frequency range. That the same dynamics was observed at different wavelength might suggest that there is no characteristic length in the re-arrangement of the material structure. This confirms the assumption commonly made by previous authors [TenCate et al. (2000)] that the changes occur at various scales following the probable continuous distribution of energy barriers.

C. Spatial dependence of Slow Dynamics

In the initial works on slow dynamics [TenCate and Shankland (1996); Guyer et al. (1998)], the medium was subject to a global excitation, and resulted in a global change of the sample stiffness. The excitation presented here is a local impact (as in [Lobkis and Weaver (2009)]). Determining whether the change induced by the ball impact is global (induced by the shock wave), or local is of some importance. To that end, we deploy a linear array of four sensors 10 cm away from the impact location and from the source transducer. Because the initial sample is not large enough, we use a different sample (#2) of similar composition but whose dimensions are now $1m \times 1m \times 25m$ (see Fig. 5). The array has a spatial pitch of about 10 cm. The decorrelation $K$ is found to strongly depend on the impact-receiver distance (Fig. 6). We therefore confirm the assumption issued by Lobkis and Weaver (2009): the changes induced by the ball impact are located in the vicinity of the impact, and the seismic (shock) wave emitted by the impact has negligible effect on the material stiffness at large distances.

V. TENTATIVE APPLICATION TO ON-SITE DAMAGE ESTIMATION

Last, we address the issue of damage estimation through DAWS. Consider two blocks of concrete of similar constitution (see tab. I) and of the same geometry (see sample #1): the first one is intact and the other one is damaged after undergoing a 30 MPa load. Both samples are tested using the same experimental protocol, and data processed in the same time window and frequency range. Resulting decorrelations $K_d$ are plotted in fig. 7. Similarly to Van Den Abeele et al. (2001), we observe that damage increases considerably the amplitude of change in the material, and thus the absolute decorrelation $A$ together with the speed of recovery $m$. Provided that the experimental protocol is rigorously similar from one site to another, this latter result suggests a possible route for on-site concrete damage assessment.

VI. CONCLUSION

In this article, we studied the slow dynamics behavior of concrete after a ball impact of moderate energy. We observed a drop of the acoustic velocity (probed by CWI [Poupinet et al. (1984)]) and a decorrelation of

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the diffuse waveforms (probed by DAWS). Then the material recovers over characteristic times ranging from 100 s to several 1000 s. In our experiments, the \( \log(t) \) recovery was observed at all time in the coda, and at different frequencies: this does not allow to exhibit any characteristic length, time, or energy. This feature is in agreement with a fractal structure of concrete. Slow dynamics is thus interpreted as a series of jumps of energy barriers having a continuous distribution. In principle, slow dynamics could be either due to a homogeneous change of mechanical characteristics of all constituents, or to local rearrangements of inter-grain contacts. As we observe a noticeable decorrelation of the diffuse waveforms that can not be solely ascribed to a homogeneous change of the material stiffness, we defend the second interpretation: slow dynamics is due to a collection of local changes in inter grain contacts, which simultaneously result in changes in scattering properties and in a decrease of the effective wave velocity. The rearrangements of inter grain contacts in our experiments are somehow analogous to those observed in solid friction [Lomnitz (1962); Pandit and Savage (1973); Karner and Marone (1998)]. As concrete is an example of consolidated multicomposite granular material, we also believe that our laboratory experiment shares analogy with the time evolution of shallow sedimentary geomaterials after an earthquake [Sawazaki et al. (2006); Brenguier et al. (2008)]. In our experiments, slow dynamics parameters \( A \) and \( m \) where found to be very sensitive to experimental apparatus, particularly to the impact energy and position. While keeping these features perfectly constant and the experiment perfectly reproducible, we observed that the slow dynamics parameters were also found to strongly depend on damage, which suggest that they could characterize the ageing of concrete structures. Since the experimental apparatus is light and easy to handle, it could have some practical interest in structural health monitoring, especially in civil engineering, or to perform down hole geometrical characterization, provided that a reproducible and controlled impact could be delivered to the medium at depth.

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