1. What familiar space is the $\Delta$-complex obtained from the 2-simplex $[v_1, v_2, v_3]$ by identifying the edges $[v_1, v_2]$ and $[v_2, v_3]$, with these orientations?

2. Compute the homology groups of the triangular parachute obtained from identifying the three vertices of a 2-simplex to a single point.

3. Find a structure of $\Delta$-complex describing the $n$-dimensional sphere $S^n$. Use it to compute the homology groups of $n$-dimensional spheres, and to deduce that $S^n$ and $S^m$ are not homeomorphic\(^1\) for $n \neq m$. Could we have told them apart using the fundamental group?

4. Compute the homology group of the $\Delta$-complex $X$ obtained from $\Delta^n$ by identifying all faces of the same dimension (with the orientations induced by that of $\Delta^n$). Thus $X$ has a single $k$-simplex for each $k \leq n$.

5. Take a single 3-simplex and label its vertices by 0, 1, 2 and 3. Identify the $[0, 1, 2]$ face with the $[1, 3, 0]$ face by sending the vertices 0, 1 and 2 respectively to 1, 3 and 0. Compute the homology of the resulting space. Can you recognize that space? (This is very tricky to visualize. The homology should give some indication. Finding the complex of question 1 inside that one might also help. One can also try to recognize the topology of its boundary.)

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\(^1\)Of course, at its core this proof relies of Theorem 2.2 in the lectures notes, which we have not proved.