Stabilisation of network controlled systems with a predictive approach

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• Deterministic model of the network
  – allow for non-deterministic behavior: robustness
  ⇒ use system information to increase performance

• Application to secure networks (TCP-SPX-LAN)

• Open-loop unstable system
Contents

I. Overview of Network Problems

II. Problem Formulation

III. Background on the State Predictor

IV. Computation of the Predictor’s Horizon

V. Explicit use of the Network Model
1. Overview of Network Problems

1. Quantization, encoding/decoding:
   - related to information theory,
   - control with limited information,
   - time-varying sampling,
   - differential coding - $\Delta$-Modulation.

2. Congestion and packet loss:
   - congestion control,
   - discrete analysis and game theory.

3. Link and bandwidth allocation
   - distributed systems,
   - quality of service,
   - control under communication constraints.

4. Time-delays
   - Passivity: teleoperation,
   - Stability: robustness,
   - Stochastic approach: LQG control.

$\Rightarrow$ Pole-placement: state predictor.
Modern cars:
- multiple safety/comfort devices,
- VAN/CAN,
- high jitter.

SX-29:
- open-loop unstable,
- LAN,
- high performance control.

Global Hawk (UAV):
- local + remote control,
- wideband satellite and Line-Of-Sight data link communications.

ITER:
- large multi-systems device,
- LAN: control and data signals,
- scheduled tasks.
II. Problem Formulation

- The network dynamics is described by a dynamical model,

$$\dot{z}(t) = f(z(t), u_d(t)), \quad z(t_0) = z_0$$
$$\tau(t) = h(z(t), u_d(t))$$

i.e. for secure networks (one flow) [Misra & all 00]: TCP with AQM

$$\frac{dW_i(t)}{dt} = \frac{1}{R_i(q)} - \frac{W_i(t)W_i(t - R_i(q))}{2R_i(q(t - R_i(q)))}p(t)$$
$$\frac{dq(t)}{dt} \approx -C + \sum_{i=1}^{N} \frac{W_i(t)}{R_i(q)}$$
$$R_i(q) = \frac{q}{C} + T_{pi}$$
• The remotely controlled system has the form

\[ \dot{x}(t) = Ax(t) + Bu(t - \tau(t)) \]
\[ y(t) = Cx(t) \]

• Hypotheses
  – \((A, B)\) and \((A, C)\) controllable and observable
  – the network dynamics is such that (secure network)

\[ 0 \leq \tau(t) \leq \tau_{max}, \quad \forall t \geq 0 \]
\[ \dot{\tau}(t) < 1, \quad \text{for almost all } t \geq 0 \]

NB: \(\tau(t)\) is the delay experienced by the signal, i.e. \(\dot{\tau}(t) = 1 \iff\) the data never gets to its destination.
III. Background on the State Predictor

- based on a state predictor with a time-varying horizon $\delta(t)$ [Artstein 82, Nihtilä 89, Uchida & all. 03] [Springer 2005]

\[ x(t + \delta(t)) = e^{A\delta(t)}x(t) + e^{A(t+\delta(t))} \int_{t}^{t+\delta(t)} e^{-A\theta}Bu(\theta - \tau(\theta))d\theta \]

\[ u(t) = -Kx(t + \delta(t)) \]

- results in the pole placement of the time-shifted closed-loop system

\[ \frac{dx(t + \delta(t))}{d(t + \delta(t))} = (A - BK)x(t + \delta(t)) = A_{cl} x(t + \delta(t)) \]

$\Rightarrow$ Non-linear time transformation $t \mapsto t + \delta(t)$ but exponential convergence if $A_{cl}$ Hurwitz & hyp. on $\tau(t)$ are satisfied.

- explicit use of the network dynamics: $\delta(t) = \tau(t + \delta(t))$
\[ \frac{\text{Time-Delay}}{0 \leq \tau(t) \leq \tau_{\text{max}} \quad \dot{\tau}(t) < 1} \]

**Network Model**
\[
\dot{z}(t) = f(z(t), u_d(t)), \quad z(0) = z_0 \\
\tau(t) = h(z(t), u_d(t))
\]

**Linear System**
\[
\dot{x}(t) = Ax(t) + Bu(t - \tau(t)) \\
y(t) = Cx(t)
\]

**Predictor Horizon**
\[
\delta(t) - \tau(t + \delta(t)) = 0
\]

**Time-Varying Predictive Control**
\[
u(t) = -K e^{A\delta(t)} \left[ x(t) + e^{At} \left. \int_{t}^{t+\delta(t)} e^{-A\theta} Bu(\theta - \tau(\theta)) d\theta \right] \right]
\]
IV. Computation of the Predictor's Horizon

\[ \delta(t) = \tau(t + \delta(t)) \]  

[IEEE CCA 2004]

- Let

\[ S(t) = \hat{\delta}(t) - \tau(t + \hat{\delta}(t)) \]

with

\[ \dot{S}(t) + \sigma S(t) = 0 \]

and \( \sigma > 0 \), to prevent for the numerical instabilities,

⇒ find \( \dot{\hat{\delta}}(t) \) such that \( \hat{\delta}(t) \) reaches asymptotically the manifold \( S(t) = 0 \).

Using the assumption \( \dot{\tau} \neq 1 \), \( \hat{\delta}(t) \) has the following dynamics

\[
\dot{\hat{\delta}}(t) = -\frac{\sigma}{1 - d\tau(\hat{\zeta})/d\hat{\zeta}} \hat{\delta} + \frac{d\tau(\hat{\zeta})/d\hat{\zeta} + \sigma\tau(\hat{\zeta})}{1 - d\tau(\hat{\zeta})/d\hat{\zeta}}
\]

where \( \hat{\zeta} = t + \hat{\delta} \) and \( |\epsilon(t)| = |\delta(t) - \hat{\delta}(t)| \leq \frac{|\hat{\delta}_0 - \tau(\hat{\delta}_0)| e^{-\sigma t}}{1 - \nu} \)
V. Predictor with an Estimated Horizon

\[ u(t) = -Ke^{A\delta(t)} \left[ x(t) + e^{At} \int_{t}^{t+\delta(t)} e^{-A\theta} Bu(\theta - \tau(\theta)) d\theta \right] \]

The control law writes equivalently

\[ u(t) = -Kx(t + \hat{\delta}(t)) \]

Using \( t \mapsto t + \delta(t) \):

\[ x'(t + \delta) = Ax(t + \delta) + Bu(t) \]
\[ = Ax(t + \delta) - BKx(t + \hat{\delta}) \]

which is analyzed from

\[ \Sigma_t : x'(\zeta) = (A - BK)x(\zeta) + BKA \int_{-\epsilon}^{0} x(\zeta + \theta) d\theta \]
\[ - (BK)^2 \int_{-2\epsilon}^{-\epsilon} x(\zeta + \theta) d\theta \]
**Lemma 1.** Consider the system $\Sigma_t$ with appropriate distributed initial conditions. If the following conditions hold

i) $A_{cl}$ is Hurwitz,

ii) $\epsilon(t)$ converges exponentially and is such that

$$0 < \dot{\epsilon}_M \equiv \sup_t \dot{\epsilon}(t) < \frac{1}{2}$$

then the trajectories of $x(\zeta(t))$ are asymptotically bounded.

$\Rightarrow$ $\sigma$ must be selected such that

$$\sigma < \frac{1 - \nu}{2|\hat{\delta}_0 - \tau(\hat{\delta}_0)|}$$

**Remark:** $\dot{\epsilon}_M$ is given by the precision of the network model or can be set with the transfer algorithm.
V. Explicit use of the Network Model

**Theorem 1.** Consider the system described by

\[ \dot{x}(t) = Ax(t) + Bu(t - \tau(t)) \]

where \( (A, B) \) is a controllable pair. Suppose that the delay dynamics and \( \sigma \) are such that

\[ A_1 \) \) \[ A_2 \) \[ A_3 \) \[ 0 \leq \tau(t) \leq \tau_{\text{max}}, \]
\[ \dot{\tau}(t) \leq \nu < 1, \]
\[ 0 < \dot{\epsilon}_M \equiv \sup_t \dot{\epsilon}(t) < \frac{1}{2} \]

then the state feedback control law

\[ u(t) = -Kx(t + \hat{\delta}(t)) \]
\[ \dot{\hat{\delta}}(t) = -\frac{\sigma}{1 - d\tau(\hat{\zeta})/d\hat{\zeta}} \hat{\delta} + \frac{d\tau(\hat{\zeta})/d\hat{\zeta} + \sigma \tau(\hat{\zeta})}{1 - d\tau(\hat{\zeta})/d\hat{\zeta}} \]

with \( \hat{\zeta} = \hat{\zeta}(t) = 1 + \hat{\delta}(t) \) and \( \hat{\delta}(0) = \hat{\delta}_0 \in [0, \tau_{\text{max}}] \), ensures that the closed-loop system trajectories are asymptotically stable.
Conclusions and Perspectives

• Remote stabilization via communication networks
  ⇒ stabilizing an open-loop unstable system with a time-varying delay.

• The proposed controller:
  – based on a $\delta(t)$-step ahead predictor,
  – results in an exponentially converging (non uniform) closed-loop system and pole placement on the time-shifted system,
  – applied to remote output stabilization and observer-based control,
  – robust with respect to time-delay uncertainties.

• Perspectives:
  – feedback/observer gain co-design,
  – extension to the nonlinear case,
  – investigate the network delay estimation and the dedicated network control [Briat05],
  – coupling between the system controller and the dedicated network controller.