Remote Stabilization via Time-Varying Communication Network Delays: Application to TCP networks

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• Open-loop unstable system

• Deterministic model of the network

• Application to secure networks (TCP-SPX-LAN)
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I. Background on Time-Delay


- Passivity [Anderson/Spong, Niemeyer/Slotine...]: teleoperation,

⇒ Pole-placement [Kwon/Pearson, Manitius/Olbrot...]: state predictor.

- Stability [Bo Lincoln 03, Meinsma/Zwart, Sename...]: robustness,

- Stochastic approach [Nilsson 98...]: LQG control.
II. Problem Formulation

- The transmission protocol dynamics write as

\[
\dot{z}(t) = f(z(t), u_d(t)), \quad z(t_0) = z_0 \\
\tau(t) = h(z(t), u_d(t))
\]

i.e. for secure networks (one flow)

\[
\begin{bmatrix}
W(t) \\
q(t)
\end{bmatrix} = \begin{bmatrix}
W(t) \\
p(t)
\end{bmatrix}
\]

\[
\begin{bmatrix}
u_d(t) \\
R(t)
\end{bmatrix} = \begin{bmatrix}
p(t) \\
C(t)
\end{bmatrix}
\]

\[
h(., .) \sim R(t)
\]
• The remotely controlled system has the form

\[ \dot{x}(t) = Ax(t) + Bu(t - \tau(t)) \]
\[ y(t) = Cx(t) \]

• Hypotheses

  – \((A, B)\) and \((A, C)\) controllable and observable
  – the network dynamics is such that

\[ 0 \leq \tau(t) \leq \tau_{\text{max}}, \quad \forall t \geq 0 \]
\[ \dot{\tau}(t) < 1, \quad \text{for almost all } t \geq 0 \]
III. Control design

State feedback stabilization:

- based on a state predictor with a time-varying horizon $\delta(t)$ [Artstein 82, Nihtilä 89, Uchida & all. 03]

$$x(t + \delta) = e^{A\delta}x(t) + e^{A(t+\delta)} \int_{t}^{t+\delta} e^{-A\theta} Bu(\theta - \tau(\theta)) d\theta$$

$$u(t) = -Kx(t + \delta)$$

- explicit use of the network dynamics:

$$\delta(t) = \tau(t + \delta(t))$$

- results in the pole placement of the time-shifted closed-loop system

$$\dot{x}(t + \delta(t)) = (A - BK)x(t + \delta(t)) = A_{cl} x(t + \delta(t))$$
Dynamic computation of $\delta(t) = \tau(t + \delta(t))$

- Let $S(t) = \delta(t) - \tau(t + \delta(t))$
  with $
  \dot{S}(t) + \lambda S(t) = 0$
  and $\lambda > 0$, to prevent for the numerical instabilities,

$\Rightarrow$ find $\dot{\delta}(t)$ such that $\delta(t)$ reaches asymptotically the manifold $S(t) = 0$.

Using the assumption $\dot{\tau} \neq 1$, $\delta(t)$ has the following dynamics

$$\dot{\delta}(t) = -\frac{\lambda}{1 - d\tau(\zeta)/d\zeta} \delta + \frac{d\tau(\zeta)/d\zeta + \lambda\tau(\zeta)}{1 - d\tau(\zeta)/d\zeta}$$

where $\zeta = t + \delta$. 

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\[
\dot{z}(t) = f(z(t), u_d(t)), \quad z(0) = z_0
\]
\[
\tau(t) = h(z(t), u_d(t))
\]

**Linear System**
\[
\dot{x}(t) = Ax(t) + Bu(t - \tau(t)),
\]
\[
y(t) = Cx(t)
\]

**Time-Delay**
\[
0 \leq \tau(t) \leq \tau_{max}, \quad \dot{\tau}(t) < 1
\]

**Network Model**
\[
\dot{z}(t) = f(z(t), u_d(t)), \quad z(0) = z_0
\]
\[
\tau(t) = h(z(t), u_d(t))
\]

**Predictor Horizon**
\[
\delta(t) = \tau(t + \delta(t)) = 0
\]

**Time-Varying Predictive Control**
\[
u(t) = -Ke^{A\delta(t)} \left[ x(t) + e^{At} \int_{t}^{t+\delta(t)} e^{-A\theta} Bu(\theta - \tau(\theta)) d\theta \right]
\]
Theorem (output feedback): With the previous hypotheses and 

\[ \infty > \nu(t) \geq 0, \]

\[ |\dot{\nu}(t)| < 1 \quad \forall t \]

where \( \nu(t) \) is the time-delay of the sensor channel. The observer-based feedback control law

\[
    u(t) = -K e^{A(\delta + \nu)} \hat{x}(t) - K e^{A(t+\delta)} \int_{t-\nu}^{t+\delta} e^{-A\theta} Bu(\theta - \tau(\theta)) d\theta
\]

\[
    \dot{\hat{x}}(t) = A\hat{x}(t) + Bu(t-\nu - \tau(t-\nu)) + H\{y(t-\nu) - C\hat{x}(t)\}
\]

with \( \hat{x}(t) \equiv \hat{x}(t - \nu(t)) \) ensures that the closed-loop system is bounded, and that the state \( x(t) \) converges exponentially to zero.
IV. Application: control of an inverted pendulum through a TCP network

TCP network:
From the fluid flow model developed by [Misra & all 00] and assuming that $N(\zeta)$ is known at $t$, $\delta(t)$ is obtained from

$$
\tau(\zeta) = \frac{1}{2} \left[ \frac{q(\zeta)}{C_r} + T_{pcs} \right], \quad \frac{d\tau}{d\zeta}(\zeta) = \frac{1}{2C_r} \left[ \sum_{i=1}^{N(\zeta)} \frac{W_i(\zeta)}{R_i(\zeta)} - C_r \right] \rightarrow \delta(t)
$$

T-shape ECP Inverted Pendulum:

- Dynamics: $4^{th}$ order, OL unstable, nonminimum phase, coupled nonlinearities...

- Linearized model $\rightarrow A, B$

- LQR synthesis $\rightarrow K$
Experimental setup

Network model (simulated):

\[
\begin{align*}
\frac{dW_1(t)}{dt} &= \frac{1}{R_1(t)} - \frac{W_1(t) W_1(t - R_1(t))}{2 R_1(t - R_1(t))} p_1(t), \\
\frac{dW_2(t)}{dt} &= \frac{1}{R_2(t)} - \frac{W_2(t) W_2(t - R_2(t))}{2 R_2(t - R_2(t))} p_2(t), \\
\frac{dq(t)}{dt} &= -300 + 2 \sum_{i=1}^{2} \frac{W_i(t)}{R_i(t)}, \quad q(0) = 5 \\
\tau(t) &= R_1(t)/2
\end{align*}
\]

with

\[
\begin{align*}
R_1(t) &\doteq \frac{q(t)}{300} + 0.001 \\
R_2(t) &\doteq \frac{q(t)}{300} + 0.0015 \\
p_{1,2}(t) &= 0.005 q(t - R_{1,2}(t)) \\
W_1(0) &= W_2(10) = 10 \text{ packets.}
\end{align*}
\]

Inverted Pendulum:

\[
\dot{x} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -21.54 & 0 & 14.96 & 0 \\ 0 & 0 & 0 & 1 \\ 65.28 & 0 & -15.59 & 0 \end{bmatrix} x + \begin{bmatrix} 0 \\ 8.10 \\ 0 \\ -10.31 \end{bmatrix} u(t - \tau)
\]
Experimental results

Network Behavior

Induced Delay (s)

Queue length and average TCP window size (packets)

State Predictor with Time-Varying Delay

x (m)

Q and W (packets)

force (N)

theta (deg)

Reference

Measured

w/o delay
Conclusions and Perspectives

• Remote stabilization via communication networks
  \(\Rightarrow\) stabilizing an open-loop unstable system with \(\tau(t)\).

• The proposed controller:
  – based on a \(\delta(t)\)-step ahead predictor,
  – results in an exponentially converging closed-loop system and pole placement on the time-shifted system,
  – applied to remote output stabilization and observer-based control.

• Perspectives:
  – robustness with respect to uncertainties on the time-delay (finite spectrum assignment robustness),
  – consider some more specific network features.