ASYMPTOTIC MODEL OF AN ACTIVE MIRROR

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Abstract: We state a simplified model of thin elastic plate including in the one side an embedded periodic distribution of piezoelectric sensors and actuators and in the other side a periodic distribution of rods clamped in one face of the plate and playing the role of actuators. The model is derived thanks to a version of the two-scale convergence theory introduced by one of the authors and rediscovered in (Cioranescu *et al.*, 2002). The mechanical structure under study may be encountered in the active mirrors used in new generations of telescopes.

1. INTRODUCTION

This work is motivated by the design of active mirrors encountered in new telescopes generations. The realization of a regulation law requires a simplified but precise model.

Some authors have already derived thin elastic plates models taking into account a periodic distribution of embedded piezoelectric patches that may be used as sensors or as actuators, see (Canon and Lenczner, 1998; Canon and Lenczner, 1999) for such static thin plate models. They have been derived thanks to the two-scale convergence analysed in (Allaire, 1988). The model presented here brings two contributions. In the one side it takes into account the transient effects in the plate and in the other side it includes the modelling of a periodic distribution of clamped piezoelectric rods on one of the plate faces. Let us note that in practical implantations of active mirrors, only one of the two kinds of actuators, namely the embedded patches or the rods, are used.

The derivation of the model is based on a version of the two-scale convergence developed by one of the authors, see (Lenczner and Mercier, 2004) and the reference therein, and also by (Cioranescu *et al.*, 2002) well known as the unfolding method.

The paper is organized as follows. We start by stating the thin plate model derived thanks to the asymptotic method already used in (Canon and Lenczner, 1997) but taking into account the transient effect and the clamped rods. Then, existence, uniqueness and boundness of the solution are stated. Furthermore, the homogenized model is detailed and we observe that their unknowns are only the mechanical displacements when all the other fields have been eliminated. All of them are determined in a post-processing step from the macroscopic mechanical displacements. Finally, the two-scale convergences of each field are formulated precisely.

2. STATEMENT OF THE RESULT

2.1 Statement of the problem

We consider three layers. One is purely elastic, the second one is equipped with piezoelectric inclusions which are used as actuators, the third one is equipped with piezoelectric inclusions which are used as sensors. Each cell is linked at their center with a piezoelectric rod which creates a punctual surfacic force at their center. The heterogeneities are periodically distributed. The period is denoted ε. In this paper we derive our model by making $\varepsilon \to 0$.

For the sake of simplicity, we assume that every cell contains only one piezoelectric patch per each sensor and actuator layer. Moreover we assume that every piezoelectric patch is metallized. In all the paper, the Einstein convention of summation on repeated indexes is used, with summation from one to two for Greek indexes.

Let us introduce the following notations

- ω, $X^\varepsilon = \{x_i^\varepsilon, \forall i\}$, $\Omega^\varepsilon = X^\varepsilon \times (-L, 0)$ are respectively the domain of the thin plate, the set of the centers of the i-th cell, and the union of the piezoelectric rods;
- $\omega^{1\varepsilon}$ is the union of the sensor and the actuator patches, which are assumed to be collocated;
- γ, γ_0 are respectively the boundary of ω and the part of the boundary with a strictly positive measure $meas(\gamma_0) > 0$, where the plate is clamped. We also define $\gamma_1 = \gamma \setminus \gamma_0$.
- Y is the periodicity cell. It is the union of a unit cell of the two-dimensional plate and a rod:

$$\begin{aligned} Y &= Y^P \cup Y^R \\ &= \{(-\frac{1}{2}, \frac{1}{2})^2 \times \{0\}\} \cup \{\{(0,0)\} \times (-L, 0]\}; \end{aligned}$$

- $\mathcal{S}^\varepsilon = \{S^\varepsilon\}$ is the set of the surfaces of the metallized piezoelectric patches;
- ρ^ε is the surfacic density of the plate;
- h^ε is the current source per cell-surface of the sensor layer;
- L_3^ε is the voltage distribution for the sensor layer which is constant on the metallized patch and vanishing outside;
- φ_c^ε is the voltage applied to the inclusions of the actuator layer;
- ϕ_d^ε is the voltage applied to the top of the piezoelectric rods;
- g^ε is the surface mechanical force applied to the layers $g^\varepsilon = (g_i^\varepsilon)_{i \in \{1,\ 2,\ 3\}} \in (L^2(\omega))^3$. g_3^ε takes into account the effect of the piezoelectric rods (see (7) below), and $(g_i^\varepsilon)_{i\in\{1,\ 2\}})$ are eventual other surfacic forces;
- the stiffness tensor $(Q^{ij\varepsilon})_{(i,j)\in\{1,3\}^2}$, the piezoelectric tensors $(Q^{ij\varepsilon})_{(i,j)\in\{(1,2),\ (2,3),\ (2,1),\ (3,2)\}}$ and $(G^{i\varepsilon})_{i\in\{1,2\}}$, and the permittivity coefficient $Q^{22\varepsilon}$ satisfy $\forall(i,j) \in \{1,3\}^2$,

$$\begin{aligned} Q^{ji\varepsilon}_{\alpha\beta\gamma\delta} = Q^{ij\varepsilon}_{\alpha\beta\gamma\delta} = Q^{ij\varepsilon}_{\gamma\delta\alpha\beta} = Q^{ij\varepsilon}_{\beta\alpha\delta\gamma}, \\ Q^{12\varepsilon}_{\alpha\beta} = -Q^{21\varepsilon}_{\alpha\beta},\ Q^{23\varepsilon}_{\alpha\beta} = -Q^{32\varepsilon}_{\alpha\beta}, \\ G^{1\varepsilon}_{\alpha\beta} = G^{1\varepsilon}_{\beta\alpha},\ G^{2\varepsilon}_{\alpha\beta} = G^{2\varepsilon}_{\beta\alpha}. \end{aligned}$$

Note moreover that the piezoelectric and permittivity coefficients vanish outside of the sensors inclusions. The two-dimensional model of thin clamped composite plate is

$$-\partial_\beta(Q^{11\varepsilon}_{\alpha\beta\gamma\delta}s_{\gamma\delta}(\bar{u}^\varepsilon) + Q^{12\varepsilon}_{\alpha\beta}L_3^\varepsilon + Q^{13\varepsilon}_{\alpha\beta\gamma\delta}\partial^2_{\gamma\delta}u_3^\varepsilon) = g_\alpha^\varepsilon - \partial_\beta(G^{1\varepsilon}_{\alpha\beta}\varphi_c^\varepsilon), \quad (1)$$

$$\frac{1}{\varepsilon^2}\int_{S^\varepsilon}\left(Q^{21\varepsilon}_{\gamma\delta}s_{\gamma\delta}(\bar{u}^\varepsilon) + Q^{22\varepsilon}L_3^\varepsilon + Q^{23\varepsilon}_{\gamma\delta}\partial^2_{\gamma\delta}u_3^\varepsilon\right)dx_1dx_2 = h^\varepsilon, \quad (2)$$

$$\rho^\varepsilon\partial^2_{tt}u_3^\varepsilon + \partial^2_{\alpha\beta}(Q^{31\varepsilon}_{\alpha\beta\gamma\delta}s_{\gamma\delta}(\bar{u}^\varepsilon) + Q^{32\varepsilon}_{\alpha\beta}L_3^\varepsilon + Q^{33\varepsilon}_{\alpha\beta\gamma\delta}\partial^2_{\gamma\delta}u_3^\varepsilon) = g_3^\varepsilon + \partial^2_{\alpha\beta}(G^{2\varepsilon}_{\alpha\beta}\varphi_c^\varepsilon), \quad (3)$$

where $s_{\alpha\beta}(\bar{u}^\varepsilon) = \frac{1}{2}(\partial_{x_\alpha}\bar{u}^\varepsilon_\beta + \partial_{x_\beta}\bar{u}^\varepsilon_\alpha)$, with boundary conditions

$$\begin{aligned} \bar{u}^\varepsilon_\alpha = u_3^\varepsilon = \partial_n u_3^\varepsilon = 0 \text{ on } \gamma_0, \\ (Q^{11\varepsilon}_{\alpha\beta\gamma\delta}s_{\gamma\delta}(\bar{u}^\varepsilon) + Q^{12\varepsilon}_{\alpha\beta}L_3^\varepsilon + Q^{13\varepsilon}_{\alpha\beta\gamma\delta}\partial^2_{\gamma\delta}u_3^\varepsilon)\cdot n_\beta = 0, \\ (Q^{31\varepsilon}_{\alpha\beta\gamma\delta}s_{\gamma\delta}(\bar{u}^\varepsilon) + Q^{32\varepsilon}_{\alpha\beta}L_3^\varepsilon + Q^{33\varepsilon}_{\alpha\beta\gamma\delta}\partial^2_{\gamma\delta}u_3^\varepsilon)\cdot n_\beta = 0 \text{ on } \gamma_1. \end{aligned}$$

We assume that all coefficients and imposed forces are uniformly bounded with respect to ε. The coefficients ρ^ε, Q^ε and G^ε are also assumed to be periodic. Their two-scale transforms are denoted by ρ, Q and G respectively. Moreover we assume $Q^{22\varepsilon} > 0$ and that there exists $C > 0$, satisfying, for all real symmetric matrices $\kappa^1_{\alpha\beta}$ and $\kappa^3_{\alpha\beta}$

$$\sum_{(i,j)\in\{1,3\}^2} Q^{ij\varepsilon}_{\alpha\beta\gamma\delta}\kappa^i_{\alpha\beta}\kappa^j_{\gamma\delta} \geq C \sum_{i\in\{1,3\}}\sum_{\alpha\beta}(\kappa^i_{\alpha\beta})^2.$$

For the sake of simplicity, we assume that the rods are so thin that they can be considered as one-dimensional piezoelectric bodies, and we neglect the inertial terms. Thus we consider the following system of equations posed in each rod,

$$-\partial_3(R^\varepsilon\partial_3 u_3^\varepsilon + d^\varepsilon\partial_3\varphi^\varepsilon) = 0, \quad (4)$$

$$-\partial_3(-d^\varepsilon\partial_3 u_3^\varepsilon + c^\varepsilon\partial_3\varphi^\varepsilon) = 0, \quad (5)$$

where $R^\varepsilon > 0$, d^ε and $c^\varepsilon > 0$ denote respectively the lineic stiffness coefficient, the lineic piezoelectric coefficient and the lineic permittivity coefficient of the rods. We assume that their two-scale transforms satisfy $(R^\varepsilon, d^\varepsilon, c^\varepsilon) = \varepsilon^2(R, d, c)$ where R, d, and c are three constant coefficients. The basis of the rods are assumed to be clamped with vanishing electrical potential

$$u_3^\varepsilon(x_3 = -L) = 0 \text{ and } \varphi^\varepsilon(x_3 = -L) = 0. \quad (6)$$

The surfacic forces g_3^ε on the plate results of the normal stresses at the top of the rods plus some eventual other surfacic forces $\tilde{g}_3^\varepsilon$, and their upper sides are clamped in the plate, so the displacement u_3 is continuous at the junctions, i.e.

$$\begin{gathered} g_3^\varepsilon = -(R^\varepsilon \partial_3 u_3^\varepsilon + d^\varepsilon \partial_3 \varphi^\varepsilon)\delta + \tilde{g}_3^\varepsilon, \\ u_{3|\omega}^\varepsilon(x_i^\varepsilon) = u_{3|\Omega^\varepsilon}^\varepsilon(x_i^\varepsilon, 0), \end{gathered} \tag{7}$$

where δ refers to the Dirac distribution at the attach point of each rod. Finally, an electric potential is applied on the top of the rods

$$\varphi^\varepsilon = \phi_d^\varepsilon \text{ where } x_3 = 0. \tag{8}$$

Let us introduce $H^1_{-L,0}(\Omega^\varepsilon)$ and $H^1_{0,\phi_d^\varepsilon}(\Omega^\varepsilon)$ the sets of all functions u_3^ε and φ^ε both in $H^1(\Omega^\varepsilon)$ satisfying (6) and (8). Let us define $H^2_{\gamma_0}(\omega)$ the set of all functions u_3 in $H^2(\omega)$ satisfying $u_3 = \partial_n u_3 = 0$ on γ_0.

Let $\underline{u}_3^\varepsilon \in H^2_{\gamma_0}(\omega)$ and $\underline{u}_3^{\varepsilon 1} \in L^2(\omega)$. Let us consider the following initial conditions

$$u_3^\varepsilon(t=0) = \underline{u}_3^\varepsilon, \; \partial_t u_3^\varepsilon(t=0) = \underline{u}_3^{\varepsilon 1}. \tag{9}$$

Given $T > 0$, we assume that the following data are uniformly bounded with respect to ε:

$$\begin{gathered} \|g_\alpha^\varepsilon\|_{L^2((0,T)\times\omega)}, \; \|\tilde{g}_3^\varepsilon\|_{L^2((0,T)\times\omega)}, \\ \|\varphi_c^\varepsilon\|_{H^1((0,T);L^2(\omega^{1\varepsilon}))}, \; \varepsilon\|\phi_d^\varepsilon\|_{H^1((0,T);X^\varepsilon)}, \\ h^\varepsilon\|_{H^1((0,T);L^2(\omega^{1\varepsilon}))}, \\ \|\underline{u}_3^\varepsilon\|_{H^2(\omega)}, \; \|\underline{u}_3^{\varepsilon 1}\|_{L^2(\omega)}. \end{gathered}$$

Theorem 1. There exists $\bar{u}^\varepsilon \in C^0([0,T]; H^1_{\gamma_0}(\omega))^2$, $u_{3|\omega}^\varepsilon \in C^1([0,T]; L^2(\omega)) \cap C^0([0,T]; H^2_{\gamma_0}(\omega))$, $L_3^\varepsilon \in C^0([0,T]; L^2(\omega^{1\varepsilon}))$, $u_{3|\Omega^\varepsilon}^\varepsilon \in C^0([0,T]; H^1_{-L,0}(\Omega^\varepsilon))$, and $\varphi^\varepsilon \in C^0([0,T]; H^1_{0,\phi_d^\varepsilon}(\Omega^\varepsilon))$ solution of (1)-(5), (7), (9).

Moreover, the following norms are uniformly bounded with respect to ε,

$$\begin{gathered} \|\bar{u}^\varepsilon\|_{L^\infty((0,T);H^1(\omega))^2}, \; \|u_{3|\omega}^\varepsilon\|_{L^\infty((0,T);H^2(\omega))}, \\ \|\partial_t u_{3|\omega}^\varepsilon\|_{L^\infty((0,T);L^2(\omega))}, \; \varepsilon\|u_{3|\Omega^\varepsilon}^\varepsilon\|_{L^\infty((0,T);H^1(\Omega^\varepsilon))}, \\ \|L_3^\varepsilon\|_{L^\infty((0,T);L^2(\omega^{1\varepsilon}))}, \; \varepsilon\|\varphi^\varepsilon\|_{L^\infty((0,T);H^1(\Omega^\varepsilon))}. \end{gathered}$$

2.1.1. Definition of the microscopic problem
For given real symmetric matrices $\kappa^1_{\gamma\delta}$, $\kappa^2_{\gamma\delta}$ and real numbers $\varphi_c^\star$, $\phi_d^\star$, ζ, let us consider the microscopic problem:
Find $(\bar{w}, K_3, w_3^0, w_3^2, \chi) \in V^1_{ad}(\zeta, \phi_d^\star)$ solution of

$$\begin{gathered} a^{1P}((\bar{w}, K_3, w_3^2), (\bar{v}^1, \tilde{L}_3^0, v_3^2)) = b^1(\bar{v}^1, \tilde{L}_3^0, v_3^2), \\ a^{1R}((w_3^0, \chi), (v_3^0, \psi)) = 0, \end{gathered}$$

for all $(\bar{v}^1, \tilde{L}_3^0, v_3^0, v_3^2, \psi) \in V^1_{ad}(0,0)$ with

$$a^{1P}((\bar{w}, K_3, w_3^2), (\bar{v}^1, \tilde{L}_3^0, v_3^2))$$
$$= \int_{Y^P} (S(\bar{v}^1), \tilde{L}_3^0, \nabla_{\hat{y}}\nabla_{\hat{y}} v_3^2) Q \begin{pmatrix} S(\bar{w}), \\ K_3 \\ \nabla_{\hat{y}}\nabla_{\hat{y}} w_3^2 \end{pmatrix} d\hat{y},$$

where $S(\bar{v}^1) = (S_{\alpha\beta}(\bar{v}^1))_{\alpha\beta} = (\frac{1}{2}(\partial_{y_\alpha} u_\beta + \partial_{y_\beta} u_\alpha))_{\alpha\beta}$,

$$a^{1R}((w_3^0, \chi), (v_3^0, \psi))$$
$$= \int_{Y^R} (\partial_3 v_3^0, \partial_3 \psi) \begin{pmatrix} R & d \\ -d & c \end{pmatrix} \begin{pmatrix} \partial_3 w_3^0 \\ \partial_3 \chi \end{pmatrix} dy_3,$$

$$b^1(\bar{v}^1, \tilde{L}_3^0, v_3^2)$$
$$= -\int_{Y^P} (S(\bar{v}^1), \tilde{L}_3^0, \nabla_{\hat{y}}\nabla_{\hat{y}} v_3^2) Q \begin{pmatrix} \kappa^1 \\ 0 \\ \kappa^2 \end{pmatrix} d\hat{y}$$
$$+\varphi_c^\star \int_{Y^P} (S(\bar{v}^1), \nabla_{\hat{y}}\nabla_{\hat{y}} v_3^2) \begin{pmatrix} G^1 \\ G^2 \end{pmatrix} d\hat{y},$$

and $V^1_{ad}(\zeta, \phi_d^\star) = \{(\bar{v}^1, \tilde{L}_3^0, v_3^0, v_3^2, \psi) \in H^1_\sharp(Y^P) \times \mathbb{P}^0(Y^P)$
$\times H^1(Y^R) \times H^2_\sharp(Y^P) \times H^1(Y^R)$ s.t. $v_3^0 = \psi = 0$ on $x_3 = -L$, $(v_3^0, \psi) = (\zeta, \phi_d^\star)$ on $x_3 = 0\}$. Here $H^1_\sharp(Y^P)$ denotes the space of all functions in $H^1(\mathbb{R}^2)$ which are periodic with period Y^P, $\mathbb{P}^0(Y^P)$ is the space of all functions which are constant on the metallized patch in Y^P and vanishing otherwise, and $H^2_\#(Y^P)$ denotes the space of $u \in H^1_\#(Y^P)$ and $\nabla_y u \in H^1_\#(Y^P)$. Note that Q^{12}, Q^{22}, Q^{32} vanish outside of Y^P.

The following linear operators $\mathcal{A}$ are elaborated from the microscopic problem:

$$\begin{gathered} \begin{pmatrix} S(\bar{w}) \\ K_3 \\ \nabla_{\hat{y}}\nabla_{\hat{y}} w_3^2 \end{pmatrix} = \mathcal{A}^{P1} \begin{pmatrix} \kappa^1 \\ \kappa^2 \end{pmatrix}, \\ \begin{pmatrix} S(\bar{w}) \\ K_3 \\ \nabla_{\hat{y}}\nabla_{\hat{y}} w_3^2 \end{pmatrix} = \mathcal{A}^{P2} \varphi_c^\star, \\ \begin{pmatrix} \partial_3 w_3^0 \\ \partial_3 \chi \end{pmatrix} = \mathcal{A}^{R1} \zeta, \; \begin{pmatrix} \partial_3 w_3^0 \\ \partial_3 \chi \end{pmatrix} = \mathcal{A}^{R2} \phi_d^\star. \end{gathered}$$

Remark 1. The variables $(\zeta, w_3^0, \bar{w}, w_3^2, \chi, K_3, \varphi_c^\star, \phi_d^\star)$ refer to the variables
$(u^0_{3|Y^P}, u^0_{3|Y^R}, \bar{u}^1, u_3^2, \varphi^0, L_3^0, \varphi_c, \phi_d)$ in Theorem 2 below.

2.1.2. Definition of the homogenized coefficients
The homogenized coefficients are defined as follows:

$$Q^{PH} = \int_{Y^P} (I^{0*} + \mathcal{A}^{P1*}) Q (I^0 + \mathcal{A}^{P1}) \, d\hat{y},$$

$$Q^{RH} = \int_{Y^R} \mathcal{A}^{R1*} \begin{pmatrix} R & d \\ -d & c \end{pmatrix} \mathcal{A}^{R1} \, dy_3,$$

$$F^{PH1} = -\int_{Y^P} (I^{0*} + \mathcal{A}^{P1*}) Q \mathcal{A}^{P2} \varphi_c \, d\hat{y},$$

$$F^{PH2} = \int_{Y^P} (Id + I^1 \mathcal{A}^{P1*}) \begin{pmatrix} G^1 \varphi_c \\ 0 \\ G^2 \varphi_c \end{pmatrix} \, d\hat{y},$$

$$F^{RH} = -\int_{Y^R} \mathcal{A}^{R1*} \begin{pmatrix} R & d \\ -d & c \end{pmatrix} \mathcal{A}^{R2} \phi_d \, dy_3,$$

$$\rho^H = \int_{Y^P} \rho \, d\hat{y},$$

where (φ_c, ϕ_d) denote the two-scale limit of $(\varphi_c^\varepsilon, \phi_d^\varepsilon)$ (see Theorem 2 below), $I^0 = \begin{pmatrix} Id & 0 \\ 0 & 0 \\ 0 & Id \end{pmatrix}$, and $I^1 = \begin{pmatrix} Id & 0 \\ 0 & 0 \end{pmatrix}$.

2.1.3. The homogenized model The macroscopic behaviour of the plate is governed by the so-called homogenized model. The mechanical plate displacements are solutions of the homogenized problem:
Find $(\bar{u}^0, u_3^0) \in C^0([0,T]; H^1_{\gamma_0}(\omega))^2 \times C^1([0,T]; L^2(\omega)) \cap C^0([0,T]; H^2_{\gamma_0}(\omega))$ solution of

$$-\partial_\beta (Q^{PH11}_{\alpha\beta\gamma\delta} s_{\gamma\delta}(\bar{u}^0) + Q^{PH12}_{\alpha\beta\gamma\delta} \partial^2_{\gamma\delta} u_3^0) = g_\alpha - \partial_\beta F^{PH1}_{\alpha\beta} + \partial^2_{\gamma\delta} F^{PH2}_{\alpha\gamma\delta}, \ \forall \alpha,$$

$$\rho^H \partial^2_{tt} u_3^0 + \partial^2_{\alpha\beta} (Q^{PH21}_{\alpha\beta\gamma\delta} s_{\gamma\delta}(\bar{u}^0) + Q^{PH22}_{\alpha\beta\gamma\delta} \partial^2_{\gamma\delta} u_3^0) + Q^{RH} u_3^0 = \tilde{g}_3 + F^{RH} + \partial^2_{\alpha\beta} F^{PH2}_{3\alpha\beta},$$

$$\bar{u}^0 = u_3 = 0, \ \partial_n u_3^0 = 0 \text{ on } \gamma_0,$$

$$(Q^{PH11}_{\alpha\beta\gamma\delta} s_{\gamma\delta}(\bar{u}^0) + Q^{PH12}_{\alpha\beta\gamma\delta} \partial^2_{\gamma\delta} u_3^0) n_\beta = 0,$$

$$(Q^{PH21}_{\alpha\beta\gamma\delta} s_{\gamma\delta}(\bar{u}^0) + Q^{PH22}_{\alpha\beta\gamma\delta} \partial^2_{\gamma\delta} u_3^0) n_\beta = 0 \text{ on } \gamma_1, \ \forall \alpha,$$

$$u_3^0(t=0) = \underline{u}_3, \ \partial_t u_3^0(t=0) = \underline{u}_3^1 \text{ on } \omega$$

The distributed current field per cell, measured through the layers equipped with sensor piezoelectric patches, is equal to $\partial_t h^0$ when $L_3^0 = 0$, where L_3^0 and h^0 are the two-scale limit of L_3^ε and h^ε (see Theorem 2 below), and is given by

$$I^0(x) = \partial_t \int_{S_Y} (Q^{21}, Q^{22})(I^0 + \mathcal{A}^{P1}) \, dy_1 dy_2 \begin{pmatrix} s(\bar{u}^0) \\ \nabla_x \nabla_x u_3^0 \end{pmatrix} \text{ on } \omega, \quad (10)$$

where S_Y is the surface of the sensor patch included in Y.

Theorem 2. Let us assume the following two-scale weak convergences:

$$(\varphi_c^\varepsilon, g_\alpha^\varepsilon, \tilde{g}_3^\varepsilon) \rightharpoonup (\varphi_c, g_\alpha, \tilde{g}_3) \text{ in } L^\infty((0,T); L^2(\omega \times Y^P))^4 \text{ w}*,$$

$$(\underline{u}_3^\varepsilon, \underline{u}_3^{\varepsilon 1}) \rightharpoonup (\underline{u}_3, \underline{u}_3^1) \text{ in } L^2(\omega \times Y^P)^2 \text{ w},$$

$$\phi_d^\varepsilon \rightharpoonup \phi_d \text{ in } L^\infty((0,T); L^2(\omega \times \{0\})) \text{ w}*$$

and

$$h^\varepsilon \rightharpoonup h^0 \text{ in } L^\infty((0,T); L^2(\omega \times S_Y)) \text{ w}* .$$

Moreover we assume that $(\underline{u}_3, \underline{u}_3^1)$ does not depend on $\hat{y} \in Y^P$.

Then there exists $(\bar{u}^0, u_3^0, \bar{u}^1, u_3^2, \varphi^0, L_3^0)$ such that, up to an extraction of a subsequence, we have

- $\bar{u}^\varepsilon$, $u^\varepsilon_{3|\omega}$, $s(\bar{u}^\varepsilon)$ and $\nabla_x \nabla_x u_3^\varepsilon$ two-scale converge weakly* respectively to $\bar{u}^0$, $u^0_{3|\omega \times Y^P}$, $s(\bar{u}^0) + S(\bar{u}^1)$ and $\nabla_x \nabla_x u^0_{3|\omega \times Y^P} + \nabla_{\hat{y}} \nabla_{\hat{y}} u_3^2$ in $L^\infty((0,T); L^2(\omega \times Y^P))$;
- $u^\varepsilon_{3|\Omega^\varepsilon}$, φ^ε, $\partial_{y_3} u^\varepsilon_{3|\Omega^\varepsilon}$ and $\partial_{y_3} \varphi^\varepsilon$ two-scale converge weakly* respectively to $u^0_{3|\omega \times Y^R}$, φ^0, $\partial_{y_3} u^0_{3|\omega \times Y^R}$ and $\partial_{y_3} \varphi^0$ in $L^\infty((0,T); L^2(\omega \times Y^R))$;
- L_3^ε two-scale converges weakly* to L_3^0 in $L^\infty((0,T); L^2(\omega \times S_Y))$.

Moreover $(\bar{u}^0, u^0_{3|\omega})$ is the solution of the homogenized problem. The current given by the sensor layer of the homogenized model is given by (10).

Finally $(u^0_{3|Y^P}, u^0_{3|Y^R}, \bar{u}^1, u_3^2, \varphi^0, L_3^0)$ is the solution of the microscopic problem using the correspondence stated in Remark 1.

3. CONCLUSION

In this paper, we consider a model of a multi-layered plate with a layer which is a dielectric layer used as an actuator, an other one is used as a sensor of the vibrations, whereas the last one is a purely elastic layer. This plate is also in junction with a distribution of stacks piezoelectric actuators. We compute the asymptotic model by computing the limit of the model as the dimension of the heterogeneities tend to zero.

The stabilization problem, namely the problem of the computation of the voltage applied to the actuators such that the state tends to the equilibrium, is studied in (Le Gall *et al.*, 2006) for the bimorph mirror case. The following tracking problem should also be investigated: compute the voltage applied to the actuators in function of the output such that the state of the plate follows a prescribed trajectory.

Note finally that a preliminary work (Baudouin *et al.*, 2006) studies the design of a H_∞ controller for an adaptive optics system. The model used in (Baudouin *et al.*, 2006) is a particularization of the asymptotic model computed in the present paper.

REFERENCES

Allaire, G. (1988). Homogenization and two-scale convergence. *SIAM J. Math. Anal.* **23 (6)**, 1482-1518.

Baudouin, L., C. Prieur and D. Arzelier (2006) Robust control of a bimorph mirror for adaptive optics system. *submitted*.

Canon, E. and M. Lenczner (1997). Models of elastic plates with piezolectric inclusions. Part I: Models without homogenization. *Math. Comput. Modelling.* **26 (5)**, 79-106.

Canon, E. and M. Lenczner (1998). Deux modèles de plaques minces avec inclusions piézoélectriques et circuits électroniques distribués. *C.R. Acad. Sci. Paris.* 336, Ser. IIb, 793-798.

Canon, E. and M. Lenczner (1999). Modelling of thin elastic plates with small piezolectric inclusions and distributed electronic circuits. Models for inclusions that are small with respect to the thickness of the plate. *Journal of elasticity.* **55(11)**. 111-141.

Ciarlet, P.G. (1990). *Plates and junctions in elastic multi-structures.* Masson.

Cioranescu, D., A. Damlamian, and G. Griso (2002). Periodic unfolding and homogenization. *C.R. Acad. Sci. Paris, Ser. I.* **335**, 99-104.

Le Gall, P., C. Prieur and L. Rosier (2006) On the control of a bimorph mirror. *IFAC Control Applications of Optimisation (CAO'06).* Cachan, France.

Lenczner, M. and D. Mercier (2004). Homogenization of periodic electrical network including voltage to current amplifiers. *SIAM Journal of Multiscale Modelling and Simulation*, **2(3)**, 359-397.