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RESEARCH ARTICLE

Active vibration control of a fluid/plate system using a pole placement controller

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We consider the problem of active reduction of the structural vibrations induced by the sloshing of large masses of fuel inside a partly full tank. The proposed study focuses on an experimental device mimicking an aircraft wing made of an aluminum rectangular plate equipped with piezoelectric patches at the clamped end and with a cylindrical tip-tank, more or less filled with liquid. After deriving a representative finite dimensional model of the complete system, containing the first 5 structural modes of the plate and the first 3 liquid sloshing modes, a controller is computed. Since our main scope is to control the most energetic mode of the structure, a full state feedback method coupled with an observer is used. Finally, the controller is also tested for different initial conditions/perturbations and the results are compared with the ones obtained with an H_∞ controller. Experimental results illustrate the relevance of the chosen strategy.

Keywords: Flexible system, fluid/plate system, pole placement controller.

1 Introduction

As commercial transport aircraft designs become larger and more flexible, the impact of aeroelastic vibration of the flight dynamics increases in prominence. See e.g. Becker (2002), Raney et al. (2001) or even Kubica and Garrec (2003) for airplanes and see Shaw and Albion (1981) for helicopter dynamics. Moreover for space applications also, the interaction of flexible modes and sloshing modes disturbs the dynamics (see e.g. Bauer and Komatsu (2000) or Deyst (1969)).

For applications in aeronautics, it is crucial to suppress or to attenuate the plane wing's vibration when the wing is in interaction with the movement of the fuel inside of it (see e.g. Merten and Stephenson (1952)). Moreover, the movement of the fuel has critical impact on the stability of the plane (see Schotte and Ohayon (2009) for example) and, coupled with the rigid body motion of the aircraft, it may create uncontrollable oscillations during flight which may even lead to the destruction of the engine (see Stengel (2004)).

Smart materials are used for many applications e.g. in civil engineering. Thus flexible structures, which are equipped with piezoelectric patches, occupy a major place in the control research area. Their capability of attenuating the vibrations and measuring the deformation is described in Becker and Luber (1998), Bhikkaji et al. (2008), Fleming and Moheimani (2004) and Sun et al. (2009) among other references. In the present paper it is shown that piezoelectric devices can be useful for vibration control in aeronautics for coupled fluid/flexible structure systems.

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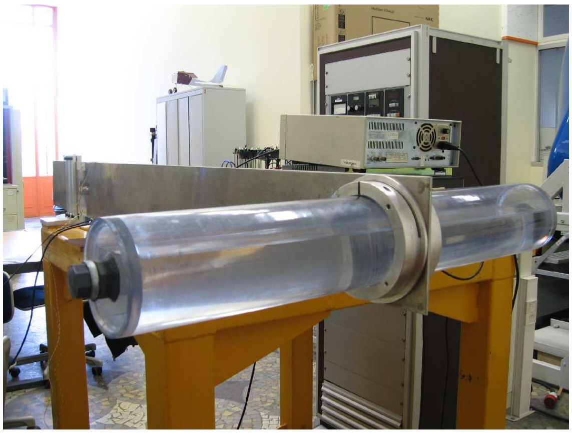

Figure 1. Plant description: the rectangular plate and the horizontal cylinder

In this objective, we study the experiment depicted by Figure 1 which is an example of a coupled fluid-flexible structure system. It has the same first flexible modes as a plane wing with fuel (see Pommier-Budinger et al. (2006)). For vibration's attenuation of flexible structures, some studies investigate the use of piezoelectric patches to effectively suppress the vibrations (see Baillargeon and Vel (2005), Denoyer and Kwak (1996), Hwang et al. (1997), Tliba (2006), Tliba et al. (2005), Yamada et al. (2007), Zhang et al. (2009) among others). However, only a few results are already available in the literature for fluid-structure systems. Reference Lasiecka and Tuffaha (2008) gives a recent theoretical result and Terasawa et al. (2004) validates the method by means of experimental results. To the best of our knowledge, there are less studies of fluid-structure system dedicated to aerospace applications, except Pommier-Budinger et al. (2008) where controllers are designed using a numerical model or our previous works Robu et al. (2009) or Robu et al. (2010), where an analytical model, described by approximated partial differential equations, is used.

In order to take into account the large number of degrees of freedom in the dynamics of the system, an infinite-dimensional model is first recalled in this paper. Afterwards, a truncation is performed in order to obtain a state-space model of the structure (see also Robu et al. (2009)). Once the model is set, a controller is computed using the pole placement method (see for example Antsaklis and Michel (2006), Godwin et al. (2001), Kuo (1967), Zadeh and Desoer (1963) among others). Although the method is well known in the literature, Matsuno et al. (2002) can be checked for a feedback control implementation on a flexible structure. One advantage of this method, besides its simplicity of implementation, is that it gives the user the possibility to choose himself the location of the closed-loop system poles, therefore allowing the possibility of placing them at some predetermined locations. The main objective is the control of the most energetic mode of the structure (e.g. the plate vibrations along the first flexion mode). After the controller implementation, we check on experiments the relevance of the proposed control strategy.

The paper is organized as follows. We present in Section 2.1 the plant under consideration, equipped with piezoelectric patches (sensors and actuators). In Section 2.2 we give a brief presentation of the analytical model and we present the control objectives. Further on in Section 3, a controller is realized using the pole placement method and it's effectiveness is shown on experiments. Finally, in Section 4, a comparison between the pole placement and a H_∞ controller is discussed for other types of initial conditions and perturbations. Section 5 gives some concluding remarks.

2 Problem statement and control objectives

2.1 *Plant description*

The plant to be controlled is located at ISAE-ENSICA, Toulouse, France and has been constructed to have the vibration frequencies of a real plane wing filled with fuel in its tip-tank (see Pommier-Budinger et al. (2006)).

The experimental device is composed of an aluminium rectangular plate and a plexiglas horizontal cylindrical tip-tank filled with liquid (see Figure 2).

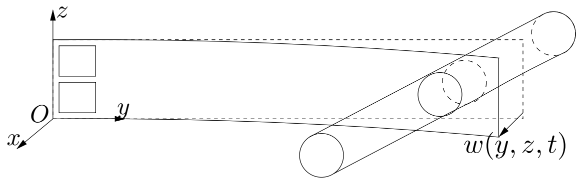

Figure 2. Deformation of the rectangular plate (1^{st} mode)

The length of the plate is along the horizontal axis and the width along the vertical one. The plate is clamped on one end and free on the three other sides. The characteristics of the aluminium plate are given in Table 1.

Plate length	L	1.36 m
Plate width	l	0.16 m
Plate thickness	h	0.005 m
Plate density	ρ	2970 kg m^{-3}
Plate Young modulus	Y	75 GPa
Plate Poisson coefficient	ν	0.33

Table 1. Plate characteristics

The two piezoelectric actuators made from PZT (Lead zirconate titanate) are bonded next to the plate clamped side. In order to create a moment, both actuators lengthen when a voltage is applied to their electrodes. Two sensors (made from PVDF - Polyvinylidene fluoride) are located on the opposite side of the plate with respect to the actuators. They will deliver a voltage proportional to their deformation. The characteristics of the collocated sensors and actuators are given in Table 2.

Actuator length/width/thickness	$0.14/0.075/5e^{-4}$ m
Sensor length/width/thickness	$0.015/0.025/5e^{-4}$ m
Actuator/Sensor density	7800 kg m^{-3}
Actuator/Sensor Young modulus	67 GPa
Actuator piezoelectric coefficient	$-210e^{-12}$ m V^{-1}
Sensor piezoelectric coefficient	-9.6 N (Vm)$^{-1}$
Actuator/Sensor Poisson coefficient	0.3

Table 2. Characteristics of the piezoelectric patches

The tank is centered at 1.28 m from the plate clamped side and is symmetrically spread along the horizontal axis. Due to the configuration of the whole system (see Figure 2), the tank undergoes a longitudinal movement when the plate has a flexion movement and a pitch movement if the plate has a torsion movement. It has the dimensions given in Table 3 and it can be filled with water or ice up to an arbitrary level. If the tank is filled with ice, it can be easily modeled by a steady mass (see Schotte and Ohayon (2009)). When the tank is filled with water up to a level close to 0 or close to the cylinder diameter (near empty tank empty or near full tank), there is no sloshing behavior, and the modeling process is similar to the case of frozen water.

Tank exterior diameter	0.11 m
Tank interior diameter	0.105 m
Tank length	0.5 m
Tank density	1180 kg m^{-3}
Tank young modulus	4.5 GPa

Table 3. Characteristics of the cylindrical tank

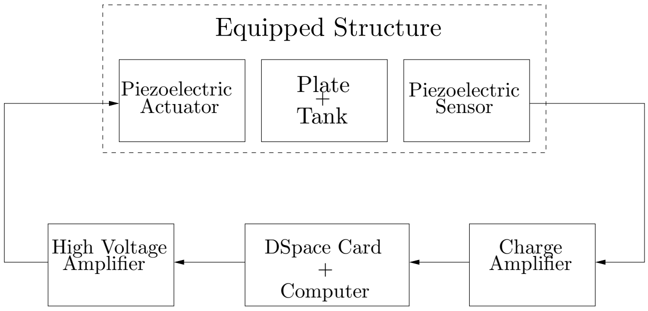

Figure 3. Equipped experimental setup

The experimental setup is depicted in Figure 3. The system (plate + tank) is connected to the computer via a high voltage amplifier delivering ± 100 V and a charge amplifier 2635 from Brüer & Kjaer. Moreover, the controller is implemented in a DSpace© board.

First, the voltage delivered by the DSpace© card is amplified by the high voltage amplifier and then applied to the piezoelectric actuator. The deflection of the beam is measured by the piezoelectric sensor and then transmitted to the charge amplifier which will deliver a voltage to the DSpace© card. Concerning the acquisition chain, tests are made using a sampling time of 0.004 seconds on the DSpace© card.

2.2 *Modeling and control objectives*

For the controller implementation, the input of the system is the voltage applied to one of the piezoelectric actuators while the output of the system is the voltage measured with a piezoelectric sensor, collocated with the control actuator.

The mathematical model of the experimental device presented above was derived in our previous work (see Robu et al. (2009)). Though the model is not detailed here, some general ideas are presented before the controller design.

Concerning the horizontal tank, it is known (Ibrahim 2005, Chapter 1) that the solution of the sloshing problem depends on the geometry of the tank. Since there are no analytical results for the mode frequencies, forces and moments of the dynamics in the horizontal cylindrical tank, a solution needs to be found.

As it was detailed in Robu et al. (2009), we use a geometrical approximation in order to overcome this difficulty. The cylindrical tank is replaced by an "equivalent" rectangular one with the same sloshing frequencies (where length and width are respectively denoted a and b and are respectively along the x-axis and y-axis) and for which analytical results are available in the literature for the calculation of modes and forces/moments (Ibrahim 2005, Chapter 1.6).

On one hand, considering the plate, we derive the partial derivative equation of the plate (see

(Géradin and Rixen 1994, Chapter 4.6)):

$$m_s \frac{\partial^2 w}{\partial t^2} + \zeta(w)\frac{\partial w}{\partial t} + YI_s\Delta^2 w = \frac{\partial^2 m_y}{\partial y^2} + \frac{\partial^2 m_z}{\partial z^2}$$

where $w = w(y, z, t)$ is the displacement, $\zeta(w)$ is an operator quantifying the damping, m_s mass per unit plate area, Y the Young modulus and $I_s = \frac{h^3}{12(1-\nu^2)}$ the moment of inertia of the plate. The Laplace operator is denoted Δ, with Δ^2 being equal to $\left(\frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right)^2$ and m_y, m_z are external moments, along the y and z-axis, delivered to the plate by the actuators (see Dimitriadis et al. (1991) or Crépeau and Prieur (2006)) and by the sloshing modes of the liquid in the tank. Furthermore the previous equation is to be solved using the boundary conditions given in (Blevins 1995, Chapter 8.1.1) for given initial conditions.

On the other hand, we now consider the longitudinal movement of the liquid along the x-axis. Since the liquid motion is starting from rest, there is a velocity potential $\phi(x, y, z, t)$ such that (see (Lamb 1995, Chapter 1.12)) the equation of liquid continuity is written as:

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} + \frac{\partial^2 \phi}{\partial z^2} = 0.$$

The linearized (Bernoulli) equation of liquid motion is given by (Lamb 1995, Chapter 2.20) or Khandelwal and Nigam (1989):

$$\frac{\partial \phi}{\partial t} + \frac{p}{\rho} + g(z - h) - C_0 x = 0$$

where C_0 stands for the acceleration along the x-axis, g for the gravitational acceleration and h for the liquid height in the container at rest position; ρ is the density of the liquid and $p = p(x, y, z, t)$ is the pressure in the liquid.

Moreover, the previous equations concerning the liquid motion are to be solved using the boundary conditions detailed in (Lamb 1995, Chapter 1.9) for the rectangular tank.

Concerning the coupling between these two equations, it is done by computing the influence of the plate on the liquid and vice-versa. More details about this issue are given in Robu et al. (2009) or Robu et al. (2010).

After a finite dimension approximation of the previous equations, we write the complete model of the structure as a state-space representation by considering only the first N modes for the plate and the first M liquid sloshing modes. Thus, the state space vector of the complete structure: $X = \begin{pmatrix} X_p & X_\theta \end{pmatrix}$ contains the state space vector of the plate X_p and the state space vector of the liquid X_θ:

$$\begin{cases} \dot{X} = \begin{pmatrix} A_p & A_{\theta p} \\ A_{p\theta} & A_\theta \end{pmatrix} X + \begin{pmatrix} B_p \\ B_{p\theta} \end{pmatrix} u \\ y = \begin{pmatrix} C_p & \mathbf{0} \end{pmatrix} X. \end{cases} \quad (1)$$

where $\begin{pmatrix} A_p & A_{\theta p} \\ A_{p\theta} & A_\theta \end{pmatrix} = A$ is the dynamic matrix of the structure, $\begin{pmatrix} B_p \\ B_{p\theta} \end{pmatrix} = B$ is the control matrix and $\begin{pmatrix} C_p & \mathbf{0} \end{pmatrix} = C$ is the output matrix with $\mathbf{0}$ denoting null matrices of appropriate dimensions.

As stated, the system model (1) is established for the first N modes of the plate and for the first M modes of the liquid sloshing. This is representative for the system behavior since it is shown in Halim and Moheimani (2003) that the first modes contain the main part of the energy of the deformation of the flexible structure. Moreover, using the energy approach from Tliba and Abou-Kandil (2003), it is possible to check that, in our particular case, the first 8 modes contain almost all the energy of the structure, with the first flexion mode of the plate containing around 65% of the total plate energy. Therefore, the controller will be computed for a system with $N = 5$ and $M = 3$. This implies $X_p \in \mathbb{R}^{10}$, $X_l \in \mathbb{R}^6$ and the total state space vector $X \in \mathbb{R}^{16}$.

The control objective is to attenuate the vibrations of the plate and the sloshing modes of the liquid in the tank for a plate initial deformation of 10cm realized at the plate free end. The deformation is along the first flexion mode of the plate, the most energetic mode. Finally, the results obtained are compared with the one given by a H_∞ control.

3 Controller synthesis

This section aims at computing a controller attenuating the plate vibrations along the first vibration mode. We are using here a state feedback strategy coupled with a Luenberger full state observer, since all the state-space vector of the system is unknown. Furthermore, based on the theory detailed in Godwin et al. (2001) or Zadeh and Desoer (1963), we use a pole placement method in order to specify the closed-loop poles and the observer poles. Furthermore, the control scheme we are following is the one depicted in Figure 4, where the matrices to be determined are K and G of suitable dimensions.

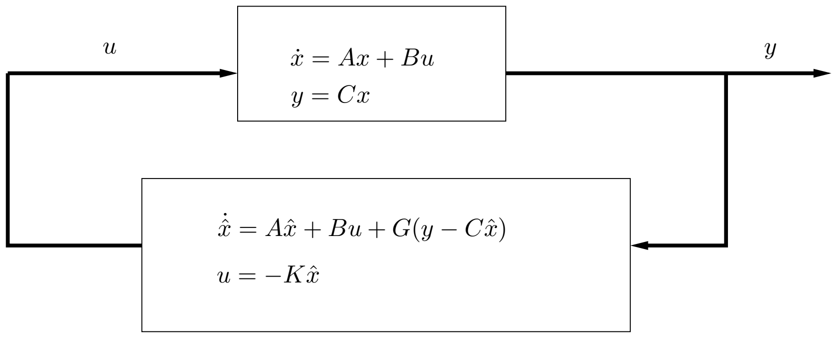

Figure 4. Feedback control law and observer

First, we compute the observability and controllability test matrices in order to be sure that all the system states are controllable and observable. Once this is set, we impose the dynamic of the state feedback law and of the observer. The poles which will specify the dynamic of the closed-loop system are chosen by selecting the poles of the $A - BK$ matrix while the ones for the observer dynamics are given by the poles of $A - GC$ matrix.

When choosing the poles one has to be very careful. In general the observer poles need to be faster than the closed-loop poles, since we want that the use of the observer does not decrease too much the performance with respect to the state feedback controller. We observed in practice that the fact of imposing very rapid poles for the observer leads to a noise amplification, thus a possible excitation of the high frequency system modes. Consequently, this will create a spillover effect, since the measurement noise is amplified. The same considerations are done for the closed-loop poles. Very fast closed-loop poles imply that: first, the voltage delivered by the controller might exceed the actuator limits of $\pm 100V$, thus possibly destabilizing the closed-loop system; second, the generated voltage might oscillate too fast in order to control the system. Thus, if the oscillating frequency is very high, the noise will be amplified, making the measurement impossible. One solution to this last issue is to select slower closed-loop poles but this will unavoidably lead to slower closed-loop response. We see therefore that a middle path needs to be found between the response time and the noise amplification.

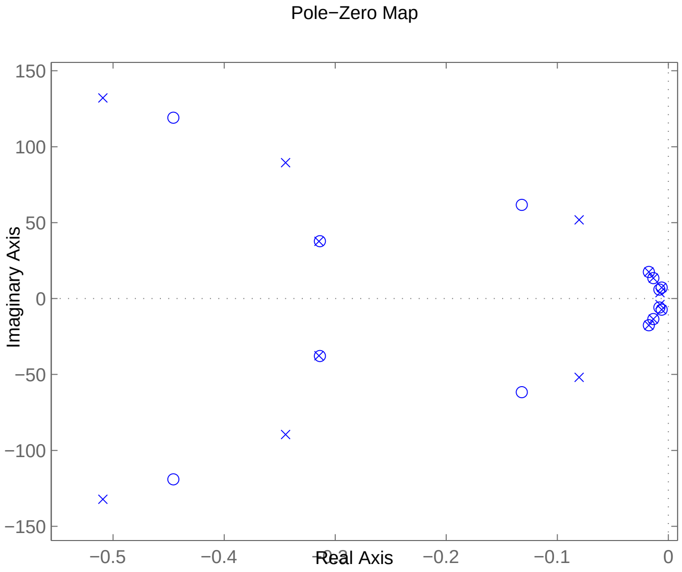

Figure 5. Pole/zero map of the open-loop system (× for the poles, ∘ for the zeros)

By checking the open-loop system poles we find 8 complex conjugate poles (3 for the liquid sloshing and 5 for the plate), all of them having their real part negative. Thus the open-loop system is stable. The position of the open-loop poles can be seen in Figure 5, while their value is presented in Table 4 below.

Since the procedure for the controller synthesis is identical for all the tank filling levels, details are given here only for the case when the tank fill level e is 0.9. Let us first consider the choice of the pole placement controller K from Figure 4.

The choice of the closed-loop poles is very difficult. The best solution is to change only the real part of the dominant poles. In this case, the best closed-loop poles are given in Table 4. Concerning all the observer poles, their real part is three times bigger than the real part of the closed-loop poles.

Open-loop poles	Closed-loop poles
-0.5093 ± 132.14i	-0.5093 ± 132.14i
-0.3447 ± 89.53i	-0.3447 ± 89.53i
-0.0803 ± 51.84i	-0.0803 ± 51.84i
-0.3146 ± 37.68i	-0.3146 ± 37.68i
-0.0175 ± 17.55i	-0.0325 ± 17.55i
-0.0135 ± 13.48i	-0.0384 ± 13.48i
-0.0059 ± 7.26i	-0.0333 ± 7.26i
-0.0074 ± 3.92i	-0.0324 ± 3.92i

Table 4. Closed-loop poles with the pole placement controller

The pole placement controller is tested on the experimental set-up for an initial plate displacement of 10cm at the free end. The controller response in attenuating the vibrations is presented in Figure 6, while the voltage delivered by the controller to make this attenuation is depicted in Figure 7.

It is important to notice that since the voltage delivered by the controller exceeds the maximum value of the voltage amplifier $\pm 100V$, the real voltage delivered to the plate in the interval $0 \ldots 30$ seconds is actually between -100V and $+100$V.

The frequency response of the system in closed-loop with the feedback controller previously computed is presented in Figure 8. We can notice that especially the first vibrations mode is well

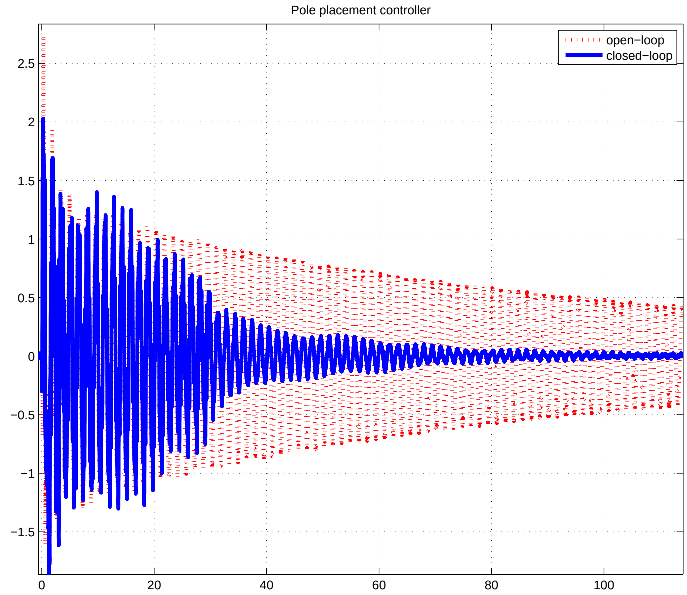

Figure 6. Experimental output of the of open-loop (dotted line) and closed-loop (plain line) systems using a pole placement controller with a tank fill level of 0.9

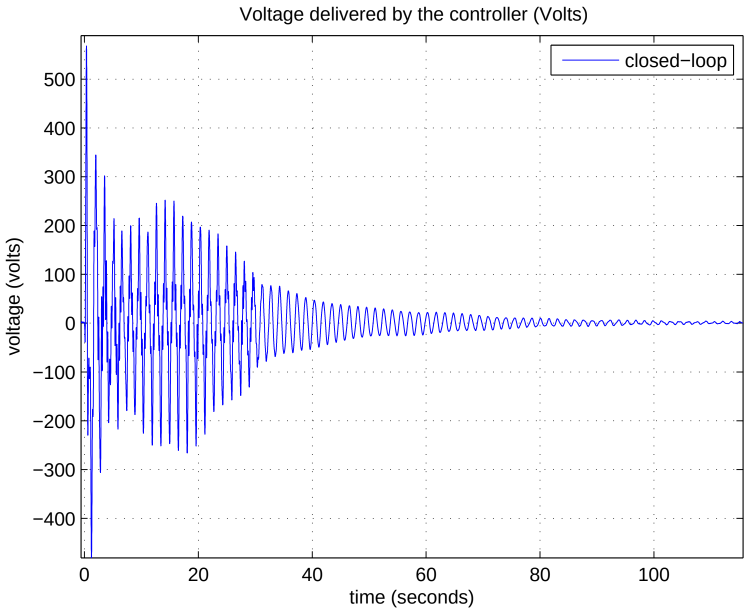

Figure 7. Voltage delivered by the pole placement controller during experiments

attenuated. Moreover, in a lower extent, the first sloshing mode (the second peak) experiences also some attenuation. We expected this to happen since the dominant poles, corresponding to the first mode of the plate and the first sloshing mode, are the ones that were mostly diminished. The other poles that were changed correspond to the other sloshing modes but their effect is not visible on the Bode plot due to their small amplitude of the concerned poles.

It is also interesting to notice that the second flexion mode also experiences a small attenuation, even though the corresponding poles have not been changed. This might be an influence of the other poles that have been shifted.

At the same time, we notice that the peaks corresponding to the torsion mode and to the other flexion modes have a larger amplitude. This means that testing the controller for a high frequency input would not give the best results since the controller is not computed to attenuate

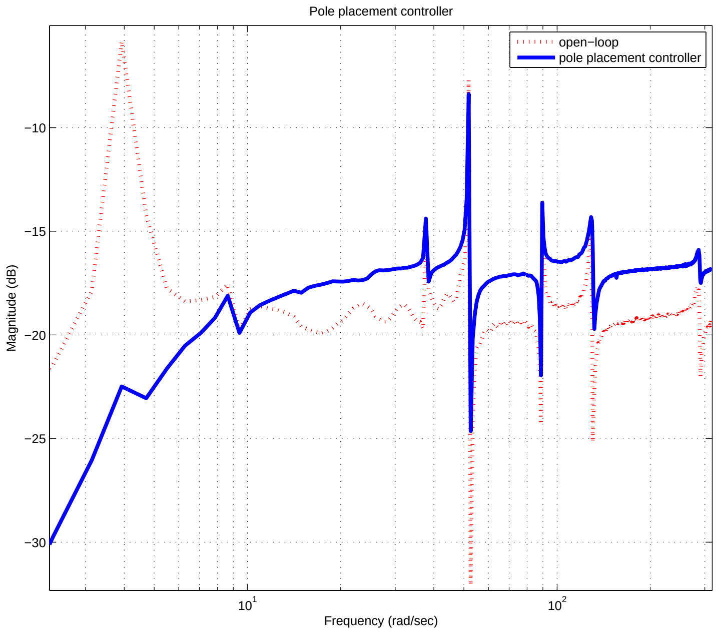

Figure 8. Frequency response of the pole placement controller

the large frequency values. Furthermore, in order to test this issue, we compare the previous results with the ones obtained using a control which has a wider action band.

4 Other control types

Another type of control was implemented on this experimental device. One can check Robu et al. (2010), where a H_∞ controller robust to external perturbations applied on the second piezoelectric actuator is computed and tested. The controller was calculated using the HIFOO library under Matlab.

In order to emphasize the advantages and the limitations of the controller calculated using the pole placement method, we compare the previous results to the ones obtained using the H_∞ controller from Robu et al. (2010). The results in temporal and frequency domains are presented below.

First of all, let us consider the case of the plate free end initial deformation of 10cm. The results are compared in Figure 9. As it can clearly be seen on experiments, the pole placement controller attenuates the plate oscillations much better than the controller computed using HIFOO does. Indeed, for the pole placement controller, the main modifications were made on the dominant poles, (see Table 4), that correspond to the first vibration and sloshing modes. Moreover, the robust controller from Robu et al. (2010) was set to minimize the influence of the perturbations on the voltage generated by the controller. Thus, the voltage generated to control the plate movements is minimized for the H_∞ case while for the pole placement case is left free. This is an explanation of the delivered voltage in both cases (around 500V for the pole placement controller and around 10V for the H_∞ one).

Now, let us consider the Bode plots of the closed-loop systems in Figure 10. It can be easily seen that even though the pole placement controller attenuates more the first flexion mode, the H_∞ controller attenuates more the other high frequency modes and even attenuates the modes that were amplified by the former controller. In spite of these issues, when considering a wider frequency band the results given by the robust controller are better overall.

Finally, a discussion can be made concerning the controllers. The chosen control strategy depends strongly on the type of problem to be solved. If one knows that the structure will

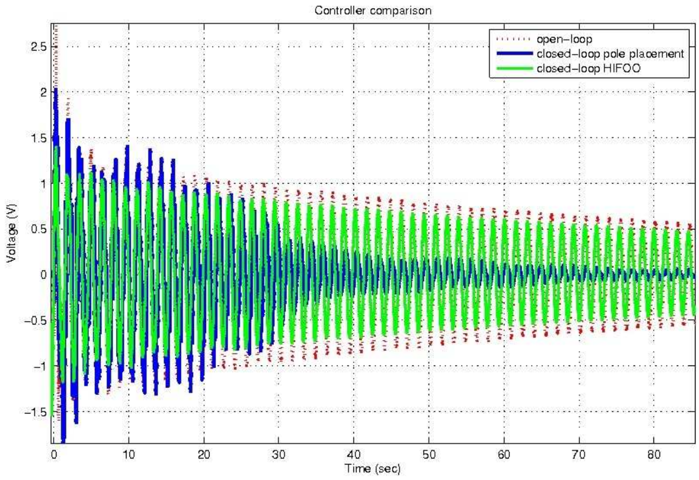

Figure 9. Temporal evolution of the experimental output for the closed-loop systems with pole placement controller (plain line) and H_∞ controller (bold line); plate free end deformation of 10cm

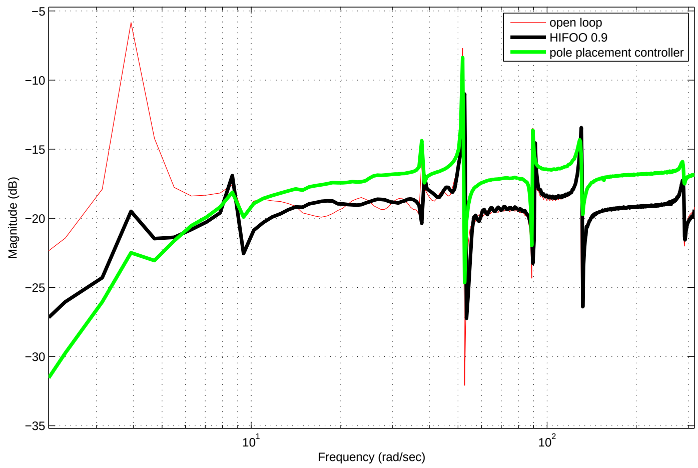

Figure 10. Experimental Bode plots for the closed-loop system with pole placement controller (plain line) and H_∞ controller (bold line)

vibrate most of the time along the first flexion mode, then the pole placement controller is clearly the best choice. On the other hand, if we consider that the frequency range in which the plate vibrates is wide, we will prefer the H_∞ controller computed using HIFOO library from Robu et al. (2010).

5 Conclusion

After describing the experimental device under consideration, some details were given concerning the computation of the analytical model. On a model with 5 vibration modes for the plate and 3 sloshing modes for the liquid in the tank, we proceeded to the control of the structure. Using the pole placement method, we computed a controller that proves to be very effective when the plate is deformed along the first flexion mode. However, the results are less conclusive when considering a wider range of frequencies. In order to better quantify these results, some experiments are performed to compare with the ones obtained with a H_∞ controller from our previous work.

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