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Stability analysis for linear systems with input backlash through sufficient LMI conditions ☆

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ABSTRACT

This paper addresses the problem of stability analysis for a given class of nonlinear systems resulting from the connection of a linear system with an isolated backlash operator. Constructive conditions based on LMIs to ensure closed-loop stability are proposed by using some suitable Lyapunov functionals with quadratic terms and Lure type terms, and generalized sector conditions. Additionally, the boundary of the associated set of all the admissible equilibrium points is precisely defined.

1. Introduction

Non-smooth nonlinearities, as hysteresis, backlash or dead-zone often occur in real process control, due to physical, technological or safety constraints, imperfections, or even inherent characteristics of considered controlled systems. A wide variety of practical systems and devices, like, for example, servo system, flexible systems, may then be represented by the interconnection of a non-smooth nonlinear operator with a plant and an actuator/controller device. In this context, there exists a large amount of literature studying control problems of nonlinear, infinite-dimensional systems resulting from a hysteresis at the input (see e.g. Barreiro and Baños (2006), Ikhouane, Mañosa and Rodellar (2005), Tao, Ma and Ling (2001), Visintin (1986), Wang and Su (2006) and Wen and Zhou (2007)). Here we focus on the stability analysis for a nonlinear system with a backlash operator (also called Krasnolsel'skii–Pokrovskii hysteresis in Macki, Nistri and Zecca (1993)). Such a nonlinearity is present in mechanical systems and its negligence during the control design or the stability analysis can lead to an important degradation of closed-loop performance or even to the loss of stability (see, in particular, Nordin, Ma and Gutman (2002)). This requires to study an infinite-dimensional system, since the backlash operator is a memory-based element. Several approaches have been developed in the context of this type of non-smooth nonlinearity; see, for example Corradini and Orlando (2002) and the references therein. In particular, one can cite the works relative to sandwich systems (Taware & Tao, 2003). Some solutions consisting in applying inverse nonlinearity have also been proposed (Taware & Tao, 2003; Taware, Tao & Teolis, 2002). Furthermore, stability and boundedness properties for systems subject to hysteresis or backlash have been studied within the scope of absolute stability (see e.g. Barreiro and Baños (2006), Haddad, Chellaboina and Oh (2000), Haddad and Kapila (1995), Logemann and Ryan (2003) and Paré, Hassibi and How (2001)). The current paper is similar in spirit to this framework and its purpose is to state a constructive method to study the stability of the nonlinear system, namely to provide an estimate of the basin of attraction around the set of equilibrium points. The approach is based on quadratic Lyapunov functionals and generalized sector conditions using the knowledge on the nonlinearity. Moreover, we need first to deal with the time-derivative version of the system to analyze the stability. This approach is not usual in the literature but is fruitful to solve the problem under consideration (see Section 3). Conditions are stated in terms of Linear Matrix Inequalities (LMI), which can be solved using standard numerical algorithms, within a time of computation which is polynomial with respect to the dimension of the data. Moreover, a bound of the set of equilibrium points, which may

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not be reduced to the origin since the backlash operator contains a dead-zone, is provided. The main contribution resides in the fact that we relax some assumptions on the static gain of the system. Actually, the symmetry of the static gain of the system is not necessary as in Paré et al. (2001); furthermore, this static gain has not to be either null or nonsingular contrary to Haddad, Chellaboina and Oh (2003) and Haddad and Kapila (1995). Hence, the conditions developed allow us to conclude about the stability of the system through a less conservative way as shown in the examples borrowed from the literature (see Section 4). Finally, let us emphasize that the results developed in this paper apply to nonstrictly proper systems (i.e., with a feedthrough term) as in Haddad et al. (2000), Haddad et al. (2003), Paré et al. (2001) and Jayawardhana, Logemann and Ryan (2008).

Notation. For two vectors x, y of $\Re^n$, the notation $x \succeq y$ means that $x_{(i)} - y_{(i)} \geq 0$, $\forall i = 1, \dots, n$. $diag(A, B)$ denotes the diagonal matrix for which the diagonal elements are the matrices A, B. $\mathbf{1}$ and $\mathbf{0}$ denote respectively the identity matrix and the null matrix of appropriate dimensions. The elements of a matrix $A \in \Re^{m\times n}$ are denoted by $A_{(i,j)}$, $i = 1, \dots, m$, $j = 1, \dots, n$. $|A|$ is the matrix constituted from the absolute value of each element of A, whereas A_+ and A_- denote the two matrices constituted with nonnegative elements such that $A = A^+ - A^-$ and $|A| = A^+ + A^-$. For two symmetric matrices, A and B, $A > B$ means that $A - B$ is positive definite. A' denotes the transpose of A. The symbol $\star$ denotes the symmetric blocks in partitioned matrices.

2. Problem formulation

Consider the following linear continuous-time system affected by a backlash operator:

$$\begin{aligned} \dot{x}(t) &= Ax(t) + B\Phi[w](t) \\ w(t) &= Cx(t) + D\Phi[w](t) \end{aligned} \tag{1}$$

where $x \in \Re^n$ is the state and $w \in \Re^m$ is the input of the nonlinearity. Here, A, B, C and D are given matrices of appropriate dimensions. For conciseness, throughout the paper, $\Phi[w](t)$ and $\dot{\Phi}[w](t)$ are denoted by $\Phi(t)$ and $\dot{\Phi}(t)$, respectively. Φ is a componentwise backlash operator studied in Brokate and Sprekels (1996), Macki et al. (1993), Paré et al. (2001) and Taware and Tao (2003). We denote the set of continuous piecewise differentiable functions $w: [0, +\infty) \to \Re^m$ by $\mathcal{C}^1_{pw}([0, +\infty); \Re^m)$, that is the set of continuous functions w being, for some unbounded sequence $(t_j)_{j=0}^{\infty}$ in $[0, +\infty)$ with $t_0 = 0$, continuously differentiable on (t_{j-1}, t_j) for all $j \in \mathbb{N}$. The operator Φ is defined as follows, for all $w \in \mathcal{C}^1_{pw}([0, +\infty); \Re^m)$, for all $j \in \mathbb{N}$, for all $t \in (t_{j-1}, t_j)$ and for all $i \in \{1, \dots, m\}$:

$$(\dot{\Phi}(t))_{(i)} = \begin{cases} \ell_{(i)}\dot{w}_{(i)}(t) & \text{if } \dot{w}_{(i)}(t) \geq 0 \\ \quad \text{and } (\Phi(t))_{(i)} = \ell_{(i)}(w_{(i)}(t) - c_{r(i)}) \\ \ell_{(i)}\dot{w}_{(i)}(t) & \text{if } \dot{w}_{(i)}(t) \leq 0 \\ \quad \text{and } (\Phi(t))_{(i)} = \ell_{(i)}(w_{(i)}(t) - c_{l(i)}) \\ 0 \quad \text{otherwise} \end{cases} \tag{2}$$

where $0 = t_0 < t_1 < \cdots$ is a partition of $[0, +\infty)$ such that w is continuously differentiable on each of the intervals (t_{j-1}, t_j), $j \in \mathbb{N}$. Each $\ell_{(i)}$, $i = 1, \dots, m$, is given in $\Re_+$, and vectors c_r and c_l are given in $\Re^m$ such that $c_l \preceq 0 \preceq c_r$. Thus, Φ is not only a nonlinearity with slope restriction as in Park, Banjerdpongchai and Kailath (1998), but also a memory-based operator, since information about the past values of w is necessary to compute it.

Remark 2.1. This definition of the backlash operator is issued from Tao and Kokotović (2002). Other equivalent definitions of a backlash operator can be written (see e.g. Brokate and Sprekels (1996), Logemann and Ryan (2003) and Logemann, Ryan and Shvartsman (2007)). As proved in Brokate and Sprekels (1996, page 42) this operator can be extended to a unique operator from the set of continuous functions to the set of continuous functions.

Assumption 2.1. Matrix A is Hurwitz.

Assumption 2.2. The static gain of system (1), that is from input Φ to output w, denoted by $G(0) = D - CA^{-1}B$, is supposed to be different from L^{-1}, where $L = \mathrm{diag}(\ell_{(1)}, \dots, \ell_{(m)})$.

Remark 2.2. Assumption 2.2 implies that matrix $\mathbf{1} + L(CA^{-1}B - D)$ is nonsingular. This is verified for the class of systems such that $\mathbf{1} + LG(s)$ is positive real (Khalil, 2002; Logemann & Ryan, 2003; Logemann et al., 2007).

Assumption 2.3. System (1) is well posed, i.e., matrix $\mathbf{1} - DL$ is supposed to be nonsingular.

Using the implicit function theorem (see Chapter 10.7 in Zorich (2004)), Assumption 2.3 allows us to verify that there exists a solution to the second equation of system (1). Hence, the results developed in what follows apply to nonstrictly proper systems (i.e., with $D \neq 0$) as in Haddad et al. (2000), Haddad et al. (2003) and Paré et al. (2001).

Throughout the paper, we consider that the nonlinearity is active (see Corradini and Orlando (2002) and Nordin et al. (2002)), that is, we are especially interested by all initial conditions $w(t = 0) = w(0)$ satisfying

$$L(w(0) - c_l) \succeq \Phi(0) \succeq L(w(0) - c_r). \tag{3}$$

Let us first remark that, with (2), one gets

$$L(w(t) - c_l) \succeq \Phi(t) \succeq L(w(t) - c_r), \quad \forall t \geq 0. \tag{4}$$

The problem we aim at solving by exploiting some properties of sector nonlinearities is a stability analysis problem which can be summarized as follows.

Problem 2.1. Characterize the set of equilibrium points toward which the trajectories of system (1)–(2) converge when initialized as in (3).

3. Stability analysis

The backlash operator Φ satisfies generalized sector conditions as stated in the lemma below. Due to the fact that Φ is a memory-based operator, it does not satisfy a classical sector condition (as described in Khalil (2002) e.g.).

Lemma 3.1. *For any diagonal positive definite matrices N_2, N_3 and positive semi-definite matrix N_1 in $\Re^{m\times m}$, with $N_3 \geq \mathbf{1}$, we have, for all $w \in \mathcal{C}^1_{pw}([0, +\infty]; \Re^m)$, for all $t \in (t_{j-1}, t_j)$*

$$\dot{\Phi}(t)'N_1(\Phi(t) - Lw(t)) \leq 0 \tag{5}$$

$$\dot{\Phi}(t)'N_2(\dot{\Phi}(t) - N_3L\dot{w}(t)) \leq 0 \tag{6}$$

where $0 = t_0 < t_1 < \cdots$ is a partition of $[0, +\infty)$ such that w is continuously differentiable on each of the intervals (t_{j-1}, t_j), $j \in \mathbb{N}$.

Proof. Let $w \in \mathcal{C}^1_{pw}([0, +\infty]; \Re^m)$, and $0 = t_0 < t_1 < \cdots$ a partition of $[0, +\infty)$ such that w is continuously differentiable on each of the intervals (t_{j-1}, t_j), $j \in \mathbb{N}$. Let $i \in \{1, \dots, m\}$. Let us start by proving (5). Consider now the three cases below.

- Assume first that $(\Phi(t))_{(i)} = \ell_{(i)}(w_{(i)}(t) - c_{r(i)})$ and $\dot{w}_{(i)}(t) > 0$, then, one gets: $(\dot{\Phi}(t)'N_1(\Phi(t) - Lw(t)))_{(i)} = -(\ell_{(i)})^2\dot{w}_{(i)}(t) N_{1(i,i)}c_{r(i)} < 0$ since $c_{r(i)} > 0$.
- Assume now that $(\Phi(t))_{(i)} = \ell_{(i)}(w_{(i)}(t) - c_{l(i)})$ and $\dot{w}_{(i)}(t) < 0$, then it follows: $(\dot{\Phi}(t)'N_1(\Phi(t) - Lw(t)))_{(i)} = -(\ell_{(i)})^2\dot{w}_{(i)}(t) N_{1(i,i)}c_{l(i)} < 0$ since $c_{l(i)} < 0$.
- Assume now that we are not in one of the previous cases. Then $(\dot{\Phi}(t))_{(i)} = 0$ and thus $(\dot{\Phi}(t)'N_1(\Phi(t) - Lw(t)))_{(i)} = 0$. This proves (5).

Let us prove (6). Let $i \in \{1, \dots, m\}$. Only two cases may occur: either $(\dot{\Phi}(t))_{(i)} = 0$, or $(\dot{\Phi}(t) - L\dot{w}(t))_{(i)} = 0$. Therefore $(\dot{\Phi}(t)'N_2(\dot{\Phi}(t) - N_3L\dot{w}(t)))_{(i)} = 0$, or $(\dot{\Phi}(t)'N_2(\dot{\Phi}(t) - N_3L\dot{w}(t)))_{(i)} = (\ell_{(i)}\dot{w}_{(i)}(t))^2 N_{2(i,i)}(1 - N_{3(i,i)}) \le 0$ provided that $1 - N_{3(i,i)} \le 0$. Thus (6) with condition $N_3 \ge \mathbf{1}$ follows. □

Since the backlash operator is defined in terms of its time-derivative, it is particularly interesting to study the time-derivative version of system (1). More precisely, by considering $X = \dot{x} \in \Re^n$ and $W = \dot{w} \in \Re^m$, the system under consideration reads

$$\begin{aligned} \dot{X}(t) &= AX(t) + B\dot{\Phi}(t) \\ W(t) &= CX(t) + D\dot{\Phi}(t) \\ w(t) &= CA^{-1}X(t) + (D - CA^{-1}B)\Phi(t). \end{aligned} \tag{7}$$

Proposition 3.1. *If there exist a symmetric positive definite matrix $P \in \Re^{n\times n}$, two diagonal positive definite matrices N_2, N and a diagonal positive semi-definite matrix N_1 in $\Re^{m\times m}$ satisfying*

$$N \ge N_2 \tag{8}$$

$$N_1 + N_1L(CA^{-1}B - D) = (CA^{-1}B - D)'LN_1 + N_1 \ge \mathbf{0} \tag{9}$$

$$\begin{pmatrix} A'P + PA & PB + C'LN + (A^{-1})'C'LN_1 \\ \star & -2N_2 + NLD + D'LN \end{pmatrix} < \mathbf{0} \tag{10}$$

then system (7) is asymptotically stable, for all initial conditions satisfying (3).

Proof. Consider the candidate Lyapunov functional $V : \Re^n \times \mathcal{C}^1_{pw}([0, +\infty); \Re^m) \times [0, +\infty) \to \Re$ defined, for all $t \in (t_{j-1}, t_j)$, by

$$\begin{aligned} V(X, \Phi, t) &= X'PX + \Phi'M\Phi \\ &- 2\int_0^t \dot{\Phi}(s)'N_1(\Phi(s) - L(CA^{-1}X(s) + (D - CA^{-1}B)\Phi(s)))\mathrm{d}s \\ &- 2\int_0^t \dot{\Phi}(s)'N_2(\dot{\Phi}(s) - N_3L(CX(s) + D\dot{\Phi}(s)))\mathrm{d}s \end{aligned} \tag{11}$$

where $0 = t_0 < t_1 < \cdots$ is a partition of $[0, +\infty)$ such that Φ is continuously differentiable on each of the intervals (t_{j-1}, t_j), $j \in \mathbb{N}$, where $P = P' > \mathbf{0}$, $M = M' \ge \mathbf{0}$, N_2, N_3 are diagonal positive definite matrices and N_1 diagonal positive semi-definite in $\Re^{m\times m}$. Due to Lemma 3.1 and relation (2), V is a positive definite functional. Let us define $M = N_1 + N_1L(CA^{-1}B - D)$ satisfying relation (9). The time-derivative of V along the solutions of (7), which is simply denoted by $\dot{V}$, reads: $\dot{V} = X'P\dot{X} + \dot{X}'PX + 2\dot{\Phi}'M\Phi - 2\dot{\Phi}'N_1(\Phi - Lw) - 2\dot{\Phi}'N_2(\dot{\Phi} - N_3LW)$ that is, with (7), $\dot{V} = X'(A'P + PA)X + 2X'(PB + C'LN_3N_2)\dot{\Phi} + \dot{\Phi}'(-2N_2 + N_2N_3LD + D'LN_3N_2)\dot{\Phi} - 2\dot{\Phi}'(N_1 + N_1L(CA^{-1}B - D))\Phi + 2\dot{\Phi}'N_1LCA^{-1}X + 2\dot{\Phi}'(N_1 + N_1L(CA^{-1}B - D))\Phi$. Thus, by setting $N = N_2N_3$ (which satisfies (8)) and $\xi = \begin{pmatrix} X' & \dot{\Phi}' \end{pmatrix}'$, one can denote: $\dot{V} = \xi'\mathcal{M}\xi$, where the matrix $\mathcal{M}$ is the left-hand term of relation (10). Hence, the satisfaction of relations (8)–(10) guarantees that one has $\dot{V} < 0$, along the trajectories of system (7). In Coron, d'Andréa Novel and Bastin (2007) it is also designed a strict Lyapunov function for a nonlinear differential equation. It directly yields the asymptotic stability property for the considered model (see also Coron (2007)). Therefore by invoking similar arguments, the closed-loop system (7) is asymptotically stable for all initial conditions satisfying (3). The proof of Proposition 3.1 is complete. □

Remark 3.1. The Lyapunov function (11), from which are based relations of Proposition 3.1, has pure quadratic terms and also Lure type terms. These latter terms use the generalized sector conditions of Lemma 3.1 based on the properties of the backlash. Without these properties, we would have only a quadratic Lyapunov function candidate (let $N_1 = N_2 = 0$ in (11)) and, following the steps of the proof of Proposition 3.1, the resulting modified condition (10) would have a null matrix in its block (2,2). Clearly, this inequality does not admit a feasible solution P. As a consequence, the knowledge of the sector conditions is essential for the stability analysis result.

Remark 3.2. Relations (8)–(10) of Proposition 3.1 are linear in the decision variables P, N_1, N_2 and N. Moreover, the relations of Proposition 3.1 do not depend on c_r and c_l. This is due to the fact that Proposition 3.1 studies the stability of the time-derivative system (7), and the variables X and $\dot{\Phi}$ do not depend on c_r and c_l. Thus, when the sufficient conditions of Proposition 3.1 hold, the variable X converges to zero (whatever the values of c_r and c_l). On the contrary, under these conditions, the original variable x converges to a point x_e which depends on c_r and c_l as shown in Theorem 3.1 below (see also Paré et al. (2001)).

A solution to Problem 2.1 is then stated below, by denoting $R = (\mathbf{1} + L(CA^{-1}B - D))^{-1}$ and $\mathcal{A} = (-A^{-1}B)$.

Theorem 3.1. *If there exist a symmetric positive definite matrix $P \in \Re^{n\times n}$, two diagonal positive definite matrices N_2, N and a diagonal positive semi-definite matrix N_1 in $\Re^{m\times m}$ such that (8)–(10) hold then, for all initial conditions satisfying (3), the trajectories of (1) converge to a point in $\mathcal{E}$ satisfying:*

$$\mathcal{E} \subset \left\{x_e \in \Re^n;\ \mathcal{A}^+\rho_1 - \mathcal{A}^-\rho_2 \succeq x_e \succeq \mathcal{A}^+\rho_2 - \mathcal{A}^-\rho_1\right\} \tag{12}$$

with $\rho_1 = R^-Lc_r - R^+Lc_l$, $\rho_2 = R^-Lc_l - R^+Lc_r$.

Proof. Consider x_e an equilibrium point of (1). Then $Ax_e + B\Phi_e = X_e = \mathbf{0}$ since the equilibrium point of system (7) verifies $X_e = \mathbf{0}$. Therefore one gets: $x_e = -A^{-1}B\Phi_e$. Furthermore, by using in relation (4) $w_e = Cx_e + D\Phi_e = (D - CA^{-1}B)\Phi_e$ one gets: $-Lc_l \succeq (\mathbf{1} + L(CA^{-1}B - D))\Phi_e \succeq -Lc_r$. The inverse of matrix $\mathbf{1} + L(CA^{-1}B - D)$ is well defined from Assumption 2.2 and is denoted by R. Then, it follows: $-R^+Lc_l + R^-Lc_r \succeq \Phi_e \succeq -R^+Lc_r + R^-Lc_l$. Hence, by multiplying the above inequality by $(-A^{-1}B)^+$ and $(-A^{-1}B)^-$ one obtains the inclusion (12). □

4. Numerical examples

4.1. Example 1

Jayawardhana, Logemann and Ryan (2008) investigated a mechanical system where the backlash operator Φ mimics a hysteretic actuator, as it can occur, for example, when using piezo-electric elements: $\mathbf{m}\ddot{x} + \mathbf{c}\dot{x} + \mathbf{k}x = \Phi[u]$, where $\mathbf{m}$ and $\mathbf{c}$ are the mass and the damping constant, $\mathbf{k}$ being associated with a linear spring constant. Considering a PD control law $u = -k_px - k_d\dot{x}$ and using the notation of system (1), system matrices are given by:

$$A = \begin{pmatrix} 0 & 1 \\ -\dfrac{\mathbf{k}}{\mathbf{m}} & -\dfrac{\mathbf{k}}{\mathbf{m}} \end{pmatrix}; \qquad B = \begin{pmatrix} 0 \\ \dfrac{1}{\mathbf{m}} \end{pmatrix};$$

$$C = \begin{pmatrix} -k_p & -k_d \end{pmatrix}; \qquad D = 0$$

with $\mathbf{m} = 1$, $\mathbf{c} = 2$, $\mathbf{k} = 4$, $k_p = 10$, $k_d = 8$, $c_r = -c_l = 0.5$, $l = 1$. Conditions stated in Theorem 3.1 are feasible. The symmetric bounds for the estimation of equilibrium set $\mathcal{E}$ are given by $\pm\begin{pmatrix} 0.0357 & 0 \end{pmatrix}'$. Fig. 1 (and its zoom around the origin) shows the time evolution of the nonlinear system for various initial conditions. One can check that the trajectories converge towards

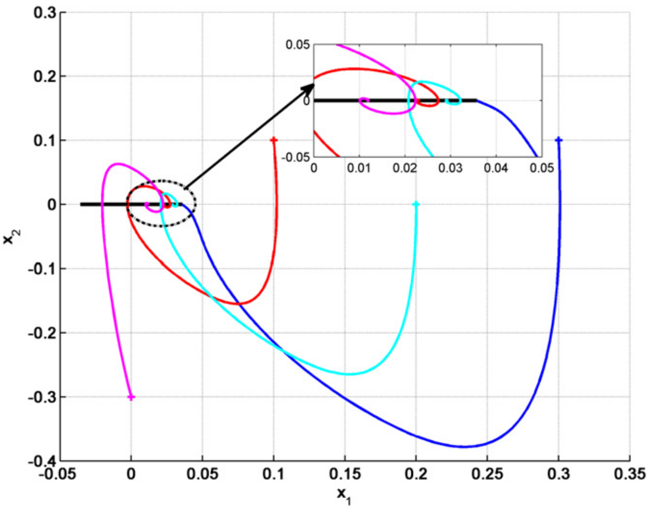

Fig. 1. Convergence of the trajectories of the nonlinear system towards the approximation of the equilibrium set $\mathcal{E}$ (line on the x_1 axis) issued from various initial conditions.

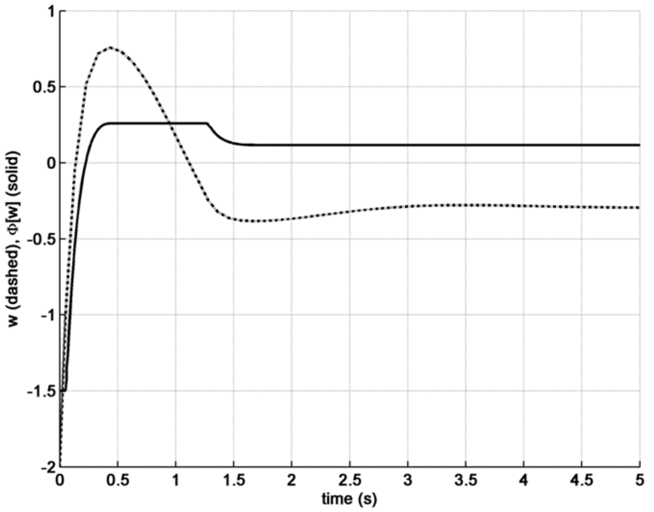

Fig. 2. Time evolution of the input (dashed line) and output (solid line) of the backlash element for a trajectory initiated in $(0.2 \quad 0)'$, with $\Phi(0) = -1.5$ satisfying (3).

non-zero equilibrium points and that the approximation of the equilibrium set is not conservative.

The input and output of the backlash operator are shown in Fig. 2, to illustrate the effect of the nonlinear element on the system.

4.2. Example 2

Let us now consider the following example borrowed from Paré et al. (2001):

$$A = \begin{bmatrix} -2 & -1 & -0.5 \\ 2 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}; \quad B = \begin{bmatrix} 0.19365 & 0.41312 \\ 0 & -0.41312 \\ 0 & 0 \end{bmatrix}$$

$$C = \begin{bmatrix} 1.875 & -0.1875 & 0.09375 \\ 1 & 0.75 & 1 \end{bmatrix}; \quad D = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$c_{r(i)} = -c_{l(i)} = 0.5; \quad l_{(i)} = 1, \ i = 1, 2.$$

Conditions stated in Theorem 3.1 are feasible and guarantee the stability of the nonlinear system, in accordance with the results of Paré et al. The symmetric bounds for the estimation of equilibrium set $\mathcal{E}$ are given by $\pm\left(0.1336 \quad 0 \quad 0.2805\right)'$, which is slightly less conservative than the values given in Paré et al. (2001), and which corresponds to their simulation results. Moreover, one can also verify that, for the system modified with $D = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}$, the stability may yet be verified thanks to Theorem 3.1. This extends the results of Paré et al., for which $G(0) = D - CA^{-1}B$ had to be symmetric.

5. Concluding remarks

The aim of the work was to analyze the stability for a class of nonlinear systems resulting from a backlash operator affecting the input of a linear plant. The main results are given in terms of constructive conditions since they are written in terms of LMIs by using some suitable Lyapunov functionals and generalized sector conditions. When dealing with such nonlinearities, there are still some open questions. The first one is the synthesis problem of the controller which will be connected to the plant through the backlash operator, by taking into account the properties of the linear plant and also those of the nonlinearity, in particular when we relax the stability assumption of matrix A. Furthermore, the case where the input of the backlash operator is nonlinear should be investigated: a first step of practical interest can be to consider nested backlash and saturation nonlinearities. Preliminary results in this sense are provided in Tarbouriech and Prieur (2007).

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