Flexible multibody dynamics: From FE formulations to control and optimization

Olivier Brüls

Multibody & Mechatronic Systems Laboratory
Department of Aerospace and Mechanical Engineering
University of Liège, Belgium

Acknowledgement to co-workers:
- Local frame methods: V. Sonneville, A. Cardona, M. Arnold
- Control: A. Lismonde, G. Bastos
- Optimization: E. Tromme

INRIA Rhône-Alpes, Grenoble, July 3, 2017
Dep. of Aerospace & Mechanical Engineering

- 23 research units (~ 130 persons)
- Research fields: aeronautics and space, solid and fluid mechanics, mechanical engineering, materials, energetics and applied maths
- Master degrees: Aerospace, Mechanics and Electromechanics
1960s: pioneering development of the FEM in the SAMCEF package
(Prof. Fraeijs de Veubeke, Prof. Sander)

1989: Extension to flexible multibody systems with MECANO (Prof. Géradin, Prof. Cardona)

1980s: Creation of Samtech (now part of Siemens PLM)

2000s: Creation of Open Engineering with the OOFELIE multiphysics package
Multibody & Mechatronic Systems Lab

Research interests

- Kinematics, dynamics & control of mechanical systems
- Specific focus on flexibility & vibrations problems
- Numerical (FE) modelling and optimization

Why the nonlinear FE approach?

- Integrated approach to represent flexible bodies with linear or nonlinear behaviour, but also rigid bodies and kinematic joints.
- Differ from the floating frame of reference technique used in standard MBS packages, in which linear elastic models are imported from an external FE software.
Outline

Introduction to our research group

More about MECANO

Local frame approach (rigid systems)

Local frame approach (flexible systems)

Optimization of MBS components

Control of flexible MBS
Example 1: wind engineering

Dynamic load prediction in a wind turbine

- Importance of flexibility effects
- Contacts and impacts in the drive-train
- Non-mechanical elements

Block diagram model

(Non-)ideal kinematic joints
Example 2: differential in a vehicle model

Torsen limited slip differential
Example 3: compliant structures

MAEVA tape spring hinge

Deployment of solar panels in a spacecraft
FE approach (Cardona 1989, Géradin & Cardona 2001)

Local frames are used to describe

- The position and orientation of a rigid body
- The position and orientation of the cross-section of a beam as a function of the centerline coordinate
- The position and orientation of the normal director of a shell as a function of the reference surface coordinates
**FE approach** (Cardona 1989, Géradin & Cardona 2001)

One local frame per node ⇒ 6 coordinates per node

- Shape functions for interpolation of translations and rotations
- Kinetic, potential, internal energies written as a function of the coordinates

Kinematic joints & rigidity conditions

- Algebraic constraints

\[
M(q)\ddot{q} + g(q, \dot{q}, t) + B^T \lambda = 0 \\
\Phi(q, t) = 0
\]

Index-3 DAE with rotation coordinates
Important technical details

- The rotation parameterization should be carefully selected as it enters the equations of motion.
- The operators behave **nonlinearly** as soon as rotations become large (even though the bodies do not deform much).
- Reduced integration is used to avoid **shear locking** problems in beam and shell formulations.
- Incremental rotation representation is used to guarantee frame invariance and avoid singularities.
- Implicit time integration method for the **index-3 DAE**.
- Scaling of equations and unknowns is necessary to avoid a **bad numerical conditioning** of the linearized problem.
- Numerical damping is needed to stabilize the constraints.
- Since the index-3 problem is solved directly (constraints at position level), **spurious but transient oscillations** appear in the initial phase.
Outline

Introduction to our research group

More about MECANO

Local frame approach (rigid systems)

Local frame approach (flexible systems)

Optimization of MBS components

Control of flexible MBS
Motivation: beyond direct analysis

Additional algorithms are needed for control design and optimization
- Optimization algorithms and sensitivity analysis
- BVP solver
- Direct transcription method
- Direct multiple shooting method
- Equivalent static load computation…

Other motivations
- Simulation interactivity (modification of B.C., loadings, etc)
- Robustness of the models w.r.t. loading, trajectory and structural parameters
- Model efficiency (e.g., for real-time control)
- Models with frictional contacts and impacts

Our goal: simplified and efficient codes which stick to the physics (we should not depend so much on the rotation parameterization)
Local frame approach

The local frame follows the motion of the body/cross section/director

The local frame is used to represent the equations of motion i.e.

- velocities and acceleration
- deformation gradients (leading to strain measures)
- forces and moments

After FE discretization, a local frame is available for each node. Actually, it represents the motion of this node.
Kinematics of a free rigid body

FE approach ⇒ one node at the CM

- One translation vector: $\mathbf{x} \in \mathbb{R}^3$
- One rotation matrix: $\mathbf{R} \in SO(3)$

The special Euclidean group $SE(3)$ is the set of $4 \times 4$ matrices

$$
H = \begin{bmatrix}
\mathbf{R} & \mathbf{x} \\
0_{1 \times 3} & 1
\end{bmatrix}
$$

with $\mathbf{R} \in SO(3)$ and $\mathbf{x} \in \mathbb{R}^3$
Kinematics of a free rigid body

\[ q = H = \begin{bmatrix} R & x \\ 0_{1 \times 3} & 1 \end{bmatrix} \]

- Composition:
  \[ q_1 q_2 = \begin{bmatrix} R_1 R_2 & x_1 + [R_1] x_2 \\ 0_{1 \times 3} & 1 \end{bmatrix} \]

- (Lie algebra) representation of velocities:
  \[ \dot{q} = \begin{bmatrix} \dot{R} & \dot{x} \\ 0_{1 \times 3} & 0 \end{bmatrix} = \begin{bmatrix} R & x \\ 0_{1 \times 3} & 1 \end{bmatrix} \begin{bmatrix} \tilde{\Omega} & U \\ 0_{1 \times 3} & 0 \end{bmatrix} \]

- Local frame velocity vector:
  \[ v = \begin{bmatrix} U \\ \Omega \end{bmatrix} \quad \text{with} \quad \begin{cases} \dot{x} = RU \\ \dot{R} = R\tilde{\Omega} \end{cases} \]
Rotating top example

- One node at the CM undergoes translations and rotations
- The fixed point condition is imposed as a constraint

\[
\mathcal{K}(\dot{x}, \Omega) = \frac{1}{2} m \dot{x}^T \dot{x} + \frac{1}{2} \Omega^T J \Omega
\]
\[
\mathcal{V}(q) = -x^T m \gamma
\]
\[
\Phi(q) = -R^T x + X = 0_{3 \times 1}
\]
\[
\mathcal{L} = \mathcal{K} - \mathcal{V}
\]
Rotating top example

\[ \Phi(q) = X - R^T x = 0_{3 \times 1} \]

Using \( \dot{x} = RU \),

\[ \dot{\Phi}\bigg|_{\Phi=0} = [-I_3 - \tilde{X}] \begin{bmatrix} U \\ \Omega \end{bmatrix} \]

Hamilton principle:

\[ \delta \int_{t_i}^{t_f} \left( \mathcal{L}(q,v) - \Phi(q)^T \lambda \right) dt = 0 \]

DAE on the special Euclidean group

- \( \dot{x} = RU \)
- \( \dot{R} = R\tilde{\Omega} \)
- \( m\dot{U} + m\Omega \times U - \lambda = R^T m\gamma \)
- \( J\dot{\Omega} + \Omega \times J\Omega + X \times \lambda = 0_{3 \times 1} \)
- \( X - R^T x = 0_{3 \times 1} \)

- Coordinate free
- Quadratic compatibility eq.
- Linear reaction forces
- Constant mass matrix
- Quadratic (but coupled) inertia forces
- Orientation-dependent gravity forces

Constant gradient

Local frame velocity
Configuration of a multibody system

\[ q = \text{diag}(H_1, \ldots, H_N, p_1, \ldots, p_M) \]

which belongs to the \( k \)-dimensional Lie group

\[ G = SE(3) \times \ldots \times SE(3) \times G_1 \times \ldots \times G_M \]

Since \( q \) needs to satisfy \( m \) kinematic constraints \( \Phi(q) \),
the configuration space is a submanifold of dimension \( k-m \)

\[ N = \{ q \in G : \Phi(q) = 0 \} \]
Equations of motion in the local frame

Index-3 DAE on a Lie group (no parameterization):

\[ \dot{q} = q\tilde{v} \]

\[ M\dot{v} - \tilde{v}^T M v = -g(q, t) - B^T(q)\lambda \]

\[ \Phi(q) = 0_{m \times 1} \]

- The configuration is described by the matrix \( q \)
- The velocity is described by a vector \( v \) and the matrix \( \tilde{v} \)
- If the initial conditions are on the group, the solution of the DAE will stay on the group for \( t \geq 0 \)
Equations of motion in the local frame

Index-3 DAE on a Lie group (no parameterization):

\[ \begin{align*}
\dot{q} &= q\tilde{v} \\
M\dot{v} - \tilde{v}^T Mv &= -g(q, t) - B^T(q)\lambda \\
\Phi(q) &= 0_{m \times 1}
\end{align*} \]

- The configuration is described by the matrix \( q \)
- The velocity is described by a vector \( v \) and the matrix \( \tilde{v} \)

Time integration on a Lie group

- Euler implicit
  \[ \begin{align*}
  q_{n+1} &= q_n \exp(h\tilde{v}_{n+1}) \\
v_{n+1} &= v_n + h\dot{v}_{n+1}
  \end{align*} \]
- Lie group generalized-\( \alpha \) method (B. and Cardona 2010, B., Cardona and Arnold 2012)
Rotating top example

Generalized-α method, $h = 0.002 \text{ s}, \rho = 0.8$

Mean number of Newton iterations

<table>
<thead>
<tr>
<th></th>
<th>Updated St</th>
<th>Frozen St</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mathbb{R}^3 \times \text{SO}(3)$</td>
<td>2.69</td>
<td>/</td>
</tr>
<tr>
<td>SE(3)</td>
<td>2</td>
<td>2.96</td>
</tr>
</tbody>
</table>

Hidden constraints are automatically satisfied by the SE(3) solution
Rotating top example

High initial velocity

Low initial velocity
Intermediate summary 1

Local frame approach (rigid systems)
- Rotations and translations are treated as a whole
- Velocities, accelerations & forces are defined in the local frame
- Rigid body constraints block the relative motion in the local frame
  \[ \Rightarrow \text{"linear" behaviour} \]
- Joint formulations only involve the relative motion
- Nonlinearities are reduced
- DAEs on a Lie group can be solved numerically
Outline

Introduction to our research group

More about MECANO

Local frame approach (rigid systems)

Local frame approach (flexible systems)

Optimization of MBS components

Control of flexible MBS
Flexible beam formulation

- Timoshenko-type geometrically exact model (cross sections do not deform)
- Translation and rotation fields \( x(s), R(s) \)
- Interpolation from nodal values \((x_A, x_B)\) and \((R_A, R_B)\)
- Strain energy: bending, torsion, traction and shear
Beam finite element formulation

Rotational & translational dofs in geometrically exact beam formulations

- Independent interpolation of rotation and translation (Simo 1985)
- Coupled interpolation using an helicoidal approximation (Borri & Bottasso, 1994)

Originality:
Formulation in the local frame

Assumption in this talk:
undehformed configuration is straight
Kinematics of the beam on $\text{SE}(3)$

\[ H(s) = \begin{bmatrix} R(s) & x(s) \\ 0_{1 \times 3} & 1 \end{bmatrix} \]
Local frame representation of strains

"Pose gradient" in the local frame

$$\frac{d(H)}{ds} = H\tilde{f}$$

$$= H(s)(\tilde{f}^0 + \tilde{\epsilon})$$

$$\epsilon = \begin{bmatrix} \gamma \\ \kappa \end{bmatrix}$$

$$\gamma = R^T(s) \frac{dx(s)}{ds} - f_U^0$$

$$\kappa = \text{vect}(R^T dR/ds)$$
Intrinsic beam formulation

\[ \frac{dH}{ds} = H(\hat{f}^0 + \tilde{\epsilon}) \quad \frac{d(H)}{dt} = H\tilde{v} \]

\[ \delta(W_{int}) = \int_0^L \delta(\epsilon)^T K \epsilon \, ds \quad \delta(K) = \int_0^L \delta(v)^T M_C v \, ds \]

Local form of the dynamic equilibrium (12-dimensional PDE)

\[ \frac{dv}{ds} = \frac{d\epsilon}{dt} + \hat{v}\epsilon \]

\[ M_C \dot{v} - \hat{v}^T M_C v + K \frac{d}{ds} (\epsilon) - \hat{f}^T K \epsilon + g_{ext} = 0 \]

Dynamic equilibrium in terms of \( f \) and \( v \) only

No need to know actual position and orientation
FE interpolation field

$$\mathbf{H}_A = \begin{bmatrix} \mathbf{R}_A & \mathbf{x}_A \\ 0_{1 \times 3} & 1 \end{bmatrix}$$

$$\mathbf{H}_B = \begin{bmatrix} \mathbf{R}_B & \mathbf{x}_B \\ 0_{1 \times 3} & 1 \end{bmatrix}$$

Interpolation on the special Euclidean group:

$$\mathbf{H}(s) = \mathbf{H}_A \exp(s \tilde{\mathbf{f}}^*) \quad \text{with} \quad \tilde{\mathbf{f}}^* = \frac{\log(\mathbf{H}_A^{-1} \mathbf{H}_B)}{L}$$
Discretized strains

Simple analytical expression of the interpolated strains

\[ \tilde{\epsilon} = \frac{\log(H_A^{-1}H_B)}{L} - \tilde{f}^0 \]

- They depend on the relative configuration between node A and B, i.e., they are invariant under rigid body motion.
- They do not depend on the coordinate along the beam.

The shape functions can thus represent exactly a constant strain field in the element.

The same observations hold for the internal forces and the tangent stiffness matrix.
No shear locking in pure bending

The interpolation field can represent exactly any helicoidal curve (constant curvature and torsion)

\[ k = \frac{M}{EI} \]

\[ x(s) = \frac{1}{k} \sin(sk) \]

\[ y(s) = \frac{1}{k} (1 - \cos(sk)) \]

\[ z(s) = 0 \]
## Static example

![Graph](image)

<table>
<thead>
<tr>
<th>Load</th>
<th>$SE(3)$ formulation</th>
<th>Simo and Vu-Quoc</th>
</tr>
</thead>
<tbody>
<tr>
<td>300</td>
<td>$\begin{array}{c} N_{it} \ x \ y \ z \end{array}$ $\begin{array}{c} 9 \ 58.84 \ 22.30 \ 40.03 \end{array}$</td>
<td>$\begin{array}{c} N_{it} \ x \ y \ z \end{array}$ $\begin{array}{c} 13 \ 58.84 \ 22.33 \ 40.08 \end{array}$</td>
</tr>
<tr>
<td>600</td>
<td>$\begin{array}{c} N_{it} \ x \ y \ z \end{array}$ $\begin{array}{c} 14 \ 47.23 \ 15.76 \ 53.28 \end{array}$</td>
<td>$\begin{array}{c} N_{it} \ x \ y \ z \end{array}$ $\begin{array}{c} 27 \ 47.23 \ 15.79 \ 53.37 \end{array}$</td>
</tr>
</tbody>
</table>
Dynamic example

This problem was solved without updating the iteration matrix in the Newton iterations.
Nonlinear shell: Example 1

360° roll-up of a clamped beam [Simo & Fox 1989]

- Poisson ratio: $\nu = 0$
- Pure bending situation
- Static solution: constant curvature

Numerical model 1

- 2 quadrangular elements
- $K_t$ is updated at each Newton iteration
- Exact solution in 1 load step

Numerical model 2

- 2 quadrangular elements
- $K_t$ is not updated
- Exact solution in 2 load steps

Numerical model 3

- 4 quadrangular elements
- $K_t$ is not updated
- Exact solution in 1 load step

No shear locking (without any numerical trick)
Nonlinear shell: example 2

Pre-twisted beam
- Linear static deformation
- 2 load cases

No shear/membrane locking (without any numerical trick)
Local frame vs. corotational frame

Local frame formulation:
- The local frame is defined from the kinematic assumption (rigid body, beam, shell).
- Several local frames may coexist in a single finite element (e.g., a beam with two nodes).
- The local frame is a nodal quantity, which is shared by all elements connected to the node (⇒ FE assembly).
- The equations of motion are written in the local frame

Corotational frame formulation:
- An additional definition is needed for the corotational frame
- The corotational frame is unique for each element
- The corotational frame is internal to the element (it is not assembled)
- The equations are finally written in the inertial frame (the corotational frame is only used at an intermediate step)
Intermediate summary 2

Local frame approach (flexible systems)

- FE formulation in the local frame
- No parameterization of the equations of motion
- For beam and shells, \( g_{int} \) and \( K_t \) are insensitive to rigid body motions
- Fluctuations of \( K_t \to 0 \) when the mesh is refined
- Finite motion problems are solved successfully without updating the tangent stiffness matrix, if the mesh is « sufficiently fine »
- No locking problem is observed (helicoidal interpolation)
- More detail: PhD thesis by Valentin Sonneville and related papers
Outline

Introduction to our research group

More about MECANO

Local frame approach (rigid systems)

Local frame approach (flexible systems)

Optimization of MBS components

Control of flexible MBS
Optimization of MBS components

Lane-change maneuver

(Virlez, 2014)
Optimization of mechanical systems

- Component-based approach
  - Experience - Empirical load case - Standard
  - Dynamic amplification
  - Not optimal wrt to the real loading
  - Iteration with the MBS team, slow and inefficient

- System-based approach
  - MBS Simulation
  - Optimization process
  - Loop
  - Time response
Fully coupled method

General and robust method

Challenges:
- Treatment of time-dependent constraints
- Sensitivity analysis of the dynamic response is costly
Weakly coupled method

Dynamic response optimization

Static response optimization problem s.t. multiple load cases
Aims at mimicking the dynamic loading

ESL definition for MBS component optimization (Kang et al., 2005):

“When a dynamic load is applied to a MBS, the equivalent static load for an isolated body is defined as the static load that produces the same relative displacement field as the one created by the dynamic load at an arbitrary time in a body-attached frame.”

General mathematical concept

At the component level, define $g_{eq}^b(t)$

Such that $K^b(p) q_{st}^b = g_{eq}^b(t)$ gives $q_{st}^b = q^b(t)$.

Number of ESLs = number of integration time steps X number of components
Weakly coupled method

Optimization of isolated components under “system” load cases

One static response optimization problem under multiple load cases

Efficient method
Generalization of the ESL method

- **Definition of the ESL at the system level**
  
  "When a dynamic load is applied to a MBS, the generalized equivalent static load is defined as the static load at the system level, that produces the same deformed configuration of the mechanism as the one created by the dynamic load at an arbitrary time."

- **Equations of motion**

  \[
  \dot{q} = q\tilde{v} \\
  M\dot{v} - \tilde{v}^T Mv = -g(q, t) - B^T(q)\lambda \\
  \Phi(q) = 0_{m \times 1}
  \]

- **System-level static load case (rigid modes are fixed)**

  \[
  g_{int} + B^T\lambda = g_{eq}(t) \\
  \Phi^*(q_{st}) = 0
  \]
Mathematical programming approach

- Optimizer: ConLin, MMA, GCM, IpOpt...
- Large displacements, material nonlinearities...
- Velocities and accelerations are not available
- Local frame formalism: constant tangent stiffness matrix!

General form of the optimization problem

\[
\begin{align*}
\text{minimize} \quad & f_0(p, q_{st}(t), \lambda(t)) \\
\text{subject to} \quad & g_{\text{int}} + B^T \lambda = g_{eq}(t) \\
& \Phi^*(q_{st}) = 0 \\
& f_j(p, q_{st}(t), \lambda(t)) \leq \bar{f}_j(t), \quad j = 1, \ldots, n_c, \\
& p_i \leq p_i \leq \bar{p}_i, \quad i = 1, \ldots, n_v.
\end{align*}
\]
Iterative scheme

1. Initial design
2. MBS simulation of the equivalent rigid body mechanism
3. MBS simulation of the flexible mechanism
4. ESL evaluation
5. Convergence check
   - Yes: Stop
   - No: Static response optimization (Inner iterations)
   - Design update: $it = it + 1$
Two dofs robot

- Imposed motion at revolute joints
- 2 beam elements per arm
- 4 design variables \( \Rightarrow \) beam diameter
- Lumped mass at point A and at the tip
Two dofs robot

(Kang, Park & Arora, 2005)

\[
\begin{align*}
\text{minimize} & \quad W(p) \\
\text{subject to} & \quad \text{Tip Trajectory Error} \leq 0.001 \text{ mm}, \\
& \quad [-75 \quad -75]^T \leq \sigma_{j,i} \leq [75 \quad 75]^T \text{ MPa}, \quad j = 1, \ldots, n_c, \\
& \quad 0.02 \text{ m} \leq p_i \leq 0.06 \text{ m}, \quad i = 1, \ldots, 4.
\end{align*}
\]

where \( n_c \) equals the number of integration time steps.
Two dofs robot

<table>
<thead>
<tr>
<th></th>
<th>$p_1, \text{mm}$</th>
<th>$p_2, \text{mm}$</th>
<th>$p_3, \text{mm}$</th>
<th>$p_4, \text{mm}$</th>
<th>$W, \text{N}$</th>
<th>MBS analysis</th>
<th>Inner iteration</th>
</tr>
</thead>
<tbody>
<tr>
<td>Proposed method</td>
<td>56.18</td>
<td>44.55</td>
<td>35.02</td>
<td>23.60</td>
<td>15.56</td>
<td>6</td>
<td>11</td>
</tr>
<tr>
<td>F. C. method</td>
<td>55.66</td>
<td>45.32</td>
<td>34.93</td>
<td>23.47</td>
<td>15.56</td>
<td>9</td>
<td>/</td>
</tr>
</tbody>
</table>
Four bar mechanism

- Impose motion at the revolute joint
- 6 beam elements per arm
- 3 design variables \( \Rightarrow \) beam diameter
Four bar mechanism

minimize \[ m(p) \]
subject to \[ \sqrt{\Delta x_A^2(t_j) + \Delta y_A^2(t_j)} \leq 0.001 \text{ m}, \quad j = 1, \ldots, 501, \]
\[ 0.015 \text{ m} \leq p_i \leq 0.5 \text{ m}, \quad i = 1, \ldots, 3, \]

The system-based ESL naturally accounts for the closed-loop conditions.
Four bar mechanism

<table>
<thead>
<tr>
<th></th>
<th>$p_1$, mm</th>
<th>$p_2$, mm</th>
<th>$p_3$, mm</th>
<th>$m$, kg</th>
<th>MBS analysis</th>
<th>Inner iteration</th>
</tr>
</thead>
<tbody>
<tr>
<td>Proposed method</td>
<td>25.717</td>
<td>15.000</td>
<td>15.000</td>
<td>1.253</td>
<td>6</td>
<td>19</td>
</tr>
<tr>
<td>F. C. method</td>
<td>25.720</td>
<td>15.000</td>
<td>15.000</td>
<td>1.253</td>
<td>7</td>
<td>/</td>
</tr>
</tbody>
</table>
Intermediate summary 3

- Optimization of mechanical systems using a system-based approach
- Equivalent static loads are defined at system level
- Local frame formalism simplifies the formulation of the equivalent static problem
- Good convergence of the weakly coupled optimization was observed in the examples

Future work:
Comparison of the efficiency between the weakly coupled method and the fully coupled method
Outline

Introduction to our research group

More about MECANO

Local frame approach (rigid systems)

Local frame approach (flexible systems)

Optimization of MBS components

Control of flexible MBS
Inverse dynamics: motivation

Direct dynamics: \( u(t) \rightarrow y(t) = ? \)
Inverse dynamics: \( y(t) \rightarrow u(t) = ? \)

Flexible systems are underactuated
Finite element approach

\( q = \) configuration variable

- collects the 3D translations & rotations of the FE nodes
- evolves a nonlinear space with a Lie group structure
- treated as a n-dimensional vector in a first step

Equations of motion - Differential Algebraic Equations (DAE)

\[
\begin{align*}
\dot{q} &= q \tilde{v} \\
M(q) \ddot{v} + f(q, v) + B^T \lambda &= A(q) u(t) \\
\Phi(q) &= 0 \\
y &= h(q)
\end{align*}
\]

Classically, it is used for simulation (forward time integration)

Here: extension to inverse dynamics computation
Inverse dynamics: formulation

For a given \( y_d(t) \), find \( u(t) \) such that

\[
\begin{align*}
\dot{q} &= q\ddot{v} \\
M(q) \ddot{v} + f(q, v) + B^T \lambda &= A(q)u \\
\Phi(q) &= 0 \\
h(q) &= y_d(t)
\end{align*}
\]

Servo constraint

(Blajer & Kolodziejczyk 2004)

This DAE can have 0, one, several or an infinite number of solutions, which are not necessarily causal.

If \( \text{dim}(u) = \text{dim}(y) \), a meaningful solution can be obtained by forward time integration of the DAE only if the internal dynamics is stable.
Forward DAE integration: example

\( w = 0.705 \text{ m} \)
\( w = 0.800 \text{ m} \)

(Seifried 2010)
DAE stable inversion

The index 3 DAE is an implicit representation of the internal dynamics

\[
\begin{align*}
\dot{q} &= q \tilde{v} \\
M(q) \dot{v} + f(q, v) + B^T \lambda &= A(q)u \\
\Phi(q) &= 0 \\
h(q) &= y_d(t)
\end{align*}
\]

- Initial conditions are relaxed ⇒ Stable inversion methods:
  - Boundary Value Problem (Devasia, Chen & Paden 1996)
  - Optimization (Bastos, Seifried & B. 2013)

- Numerical solution based on DAE methods
  - direct collocation (Bastos, Seifried & B. 2013)
  - multiple shooting (B., Bastos & Seifried 2014)
  - in both cases, generalized-α time discretization
  - extension to Lie group systems (Lismonde, Sonnevile & B. 2016)
Example

Internal dynamics trajectory

Control force $F$

$\frac{d\beta}{dt}$ [rad/s] vs $\beta$ [rad] and $t [s]$ with stable and unstable eigenspace, and control force $F$ over time with $t_0$ and $t_f$. OPT with different time steps $\Delta T$.
Parallel robot

- Parallel robot with 3 rigid dof.
- Made up of 2 tubular links (1/10 thick):
  1. **Rigid links** (3):
     Alu, 0.25 x 0.05 x 0.05 m.
  2. **Flexible links** (3 x 4 beams):
     Alu, 0.51 x 0.075 x 0.0075 m.
- **Point mass** at the end-effector (0.1 kg).
- Trajectory: **half-circle** with 0.1 m radius in the **xy plane**, to be completed in 0.6 s.
- Analysis: 1st unstable pole at 24 Hz.
Parallel robot

Velocity of joints before and after optimization

Actual output trajectories before and after optimization
Intermediate summary 4

Inverse dynamics of flexible systems

- A stable inversion is needed to obtain a bounded solution
- The FE approach leads to an implicit DAE formulation (no need to derive the I/O normal form)
- Formulation as a
  - DAE BVP on a Lie group
  - DAE optimization problem on a Lie group
- Numerical solution by
  - multiple shooting
  - direct collocation
- The method was successfully applied to the model of a 3D parallel kinematic manipulator with flexible links
Multibody & Mechatronic Systems Lab

Application

Human motion analysis
Mechatronics and robotic systems
Deployable structures

Modelling assumption

Rigid MBS
Flexible MBS
Nonsmooth flexible MBS
Mechatronic systems

Problem to solve

Performance analysis
Stress analysis
Inverse dynamics
Optimal control
Feedback control
Design opti
Merci de votre attention !

Flexible multibody dynamics:
From FE formulations to control and optimization

Olivier Brüls
Department of Aerospace and Mechanical Engineering (LTAS)
University of Liège, Belgium

INRIA Rhône-Alpes, Grenoble, July 3, 2017