Modèles numériques et géométriques pour l'image

VII. MORPHOLOGIE MATHÉMATIQUE

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objets binaires : ensemble de vecteurs correspondant aux positions des pixels objet de l'image

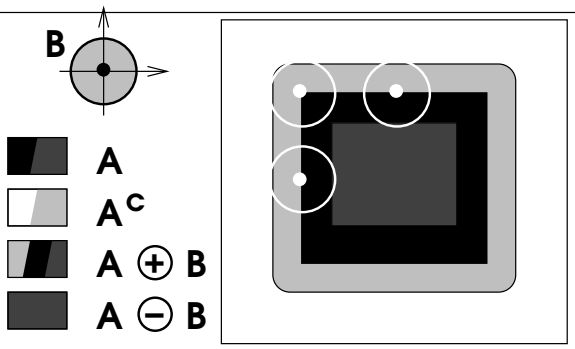
$= \{ (x,y) , f(x,y) = 1\}$

addition de Minkowski :

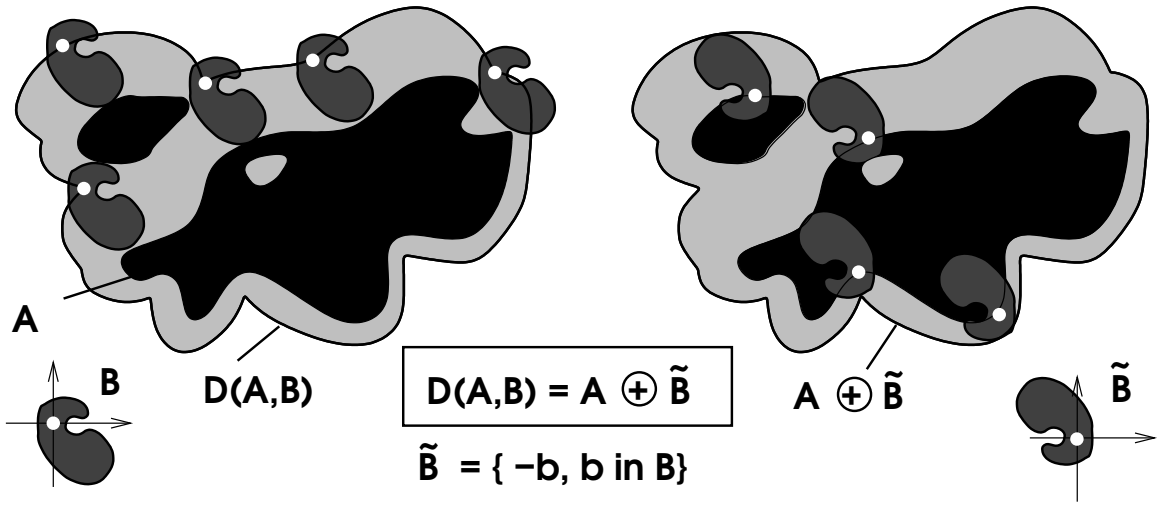
$A \oplus B = \{ a+b, a \text{ in } A, b \text{ in } B\}$

soustraction de Minkowski :

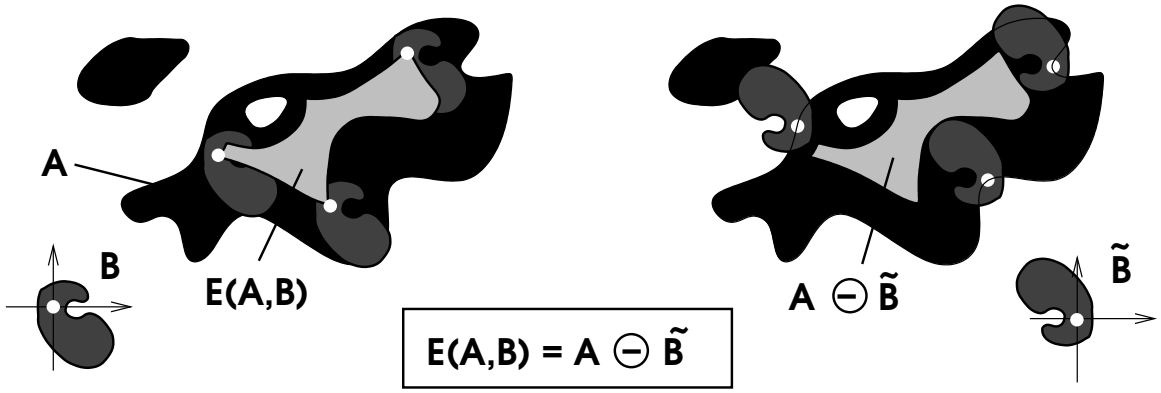
$A \ominus B = (A^c \oplus B)^c$



dilatation : $D(A,B) = \{ u, A \cap B_u \neq \emptyset \}$ $\quad B_u = \{ b+u, b \text{ in } B\}$

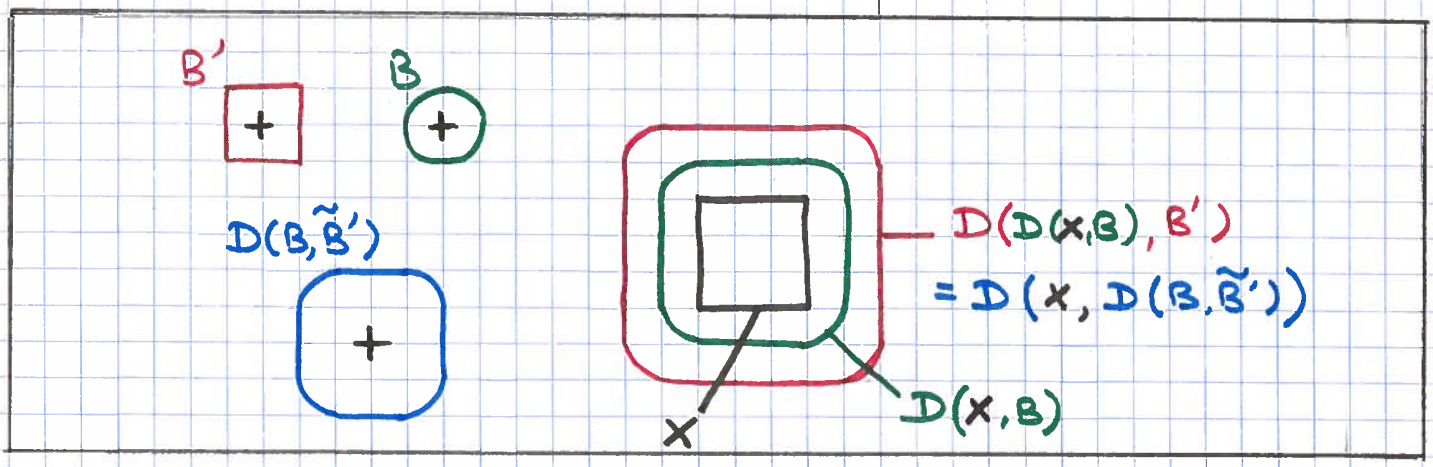


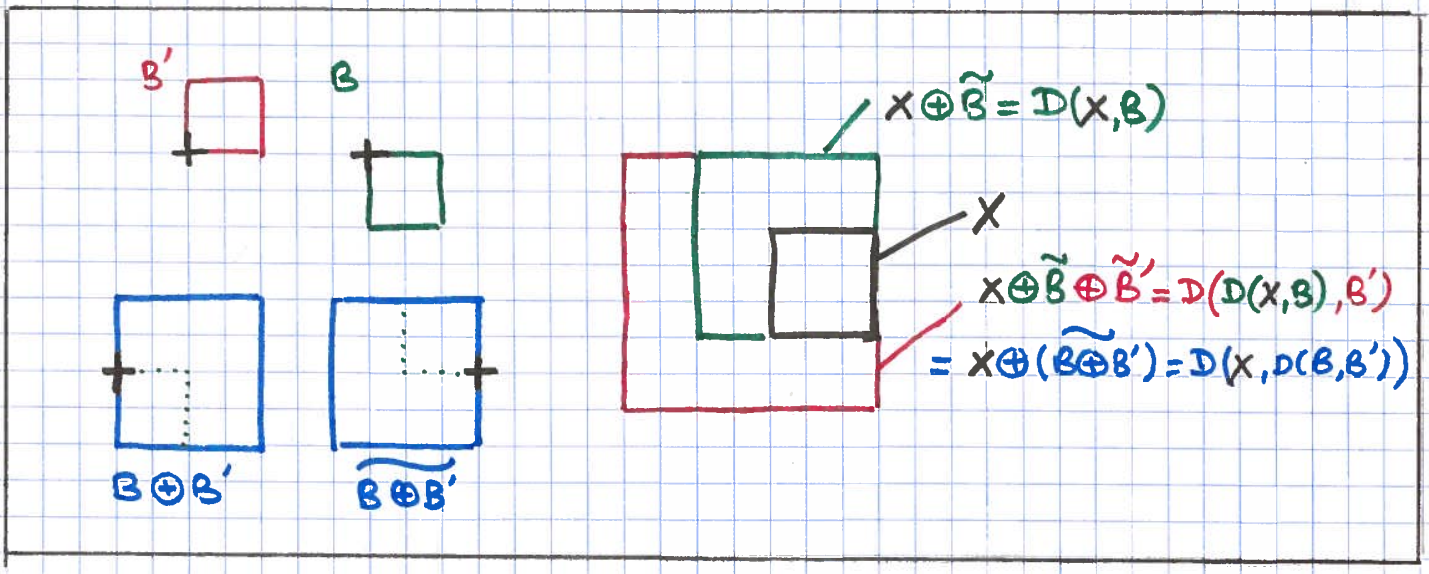
érosion : $E(A,B) = \{ u, B_u \subset A \}$



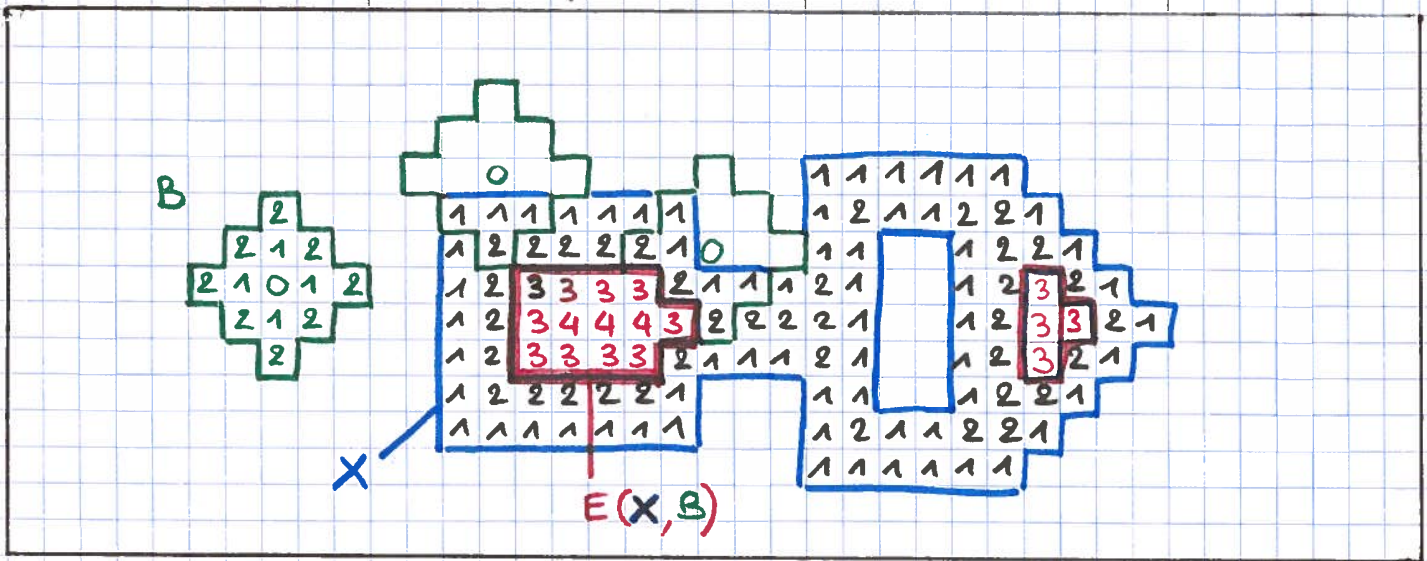
Opérateurs morphologiques sur images binaires

ASSOCIATIVITÉ DE LA DILATATION



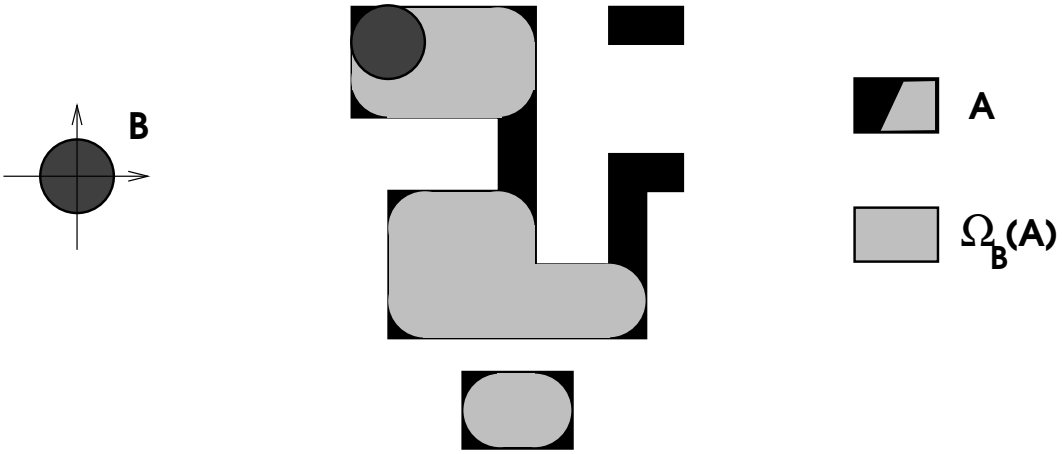


ÉROSION ET TRANSFORMÉE EN DISTANCE



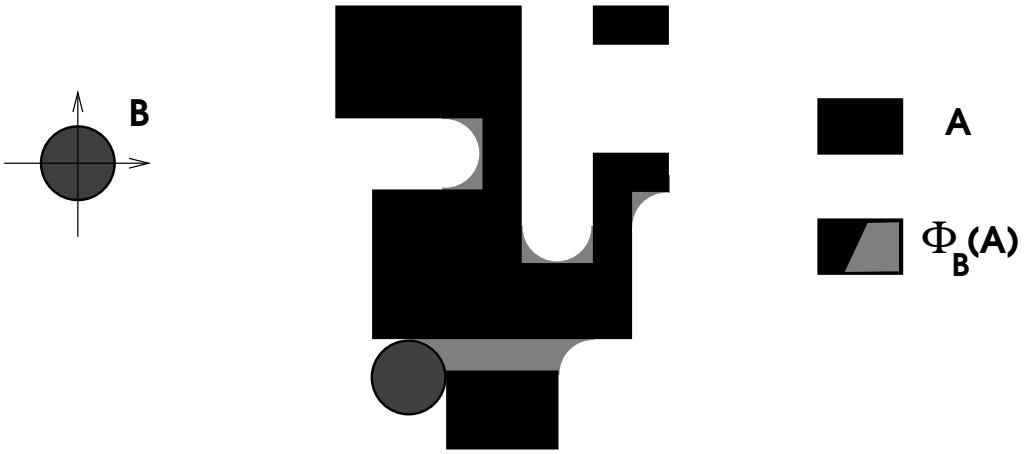
ouverture : espace balayé par l'élément structurant B lorsqu'il est inclus entièrement dans l'objet A

$$\Omega_B(A) = (A \ominus \tilde{B}) \oplus B$$



fermeture : espace non balayé par l'élément structurant B lorsqu'il est inclus entièrement dans le fond A^c

$$\Phi_B(A) = (A \oplus \tilde{B}) \ominus B$$

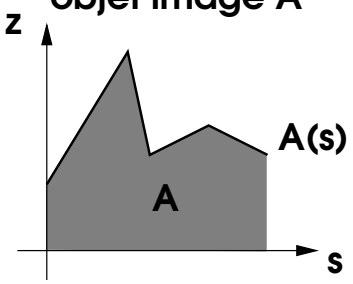


Opérateurs morphologiques sur images binaires

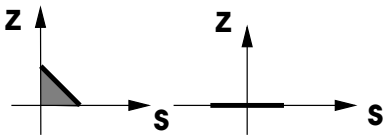
objets en niveaux de gris : sous-graphe de la fonction qui
à chaque point de l'image fait correspondre
son niveau de gris = { (x,y,z) , z <= f(x,y) }

notation : A = { (s,z), z = A(s) et s = (x,y)}

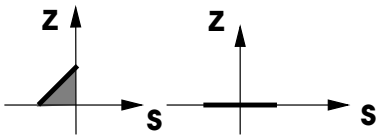
objet image A



Eléments structurants B

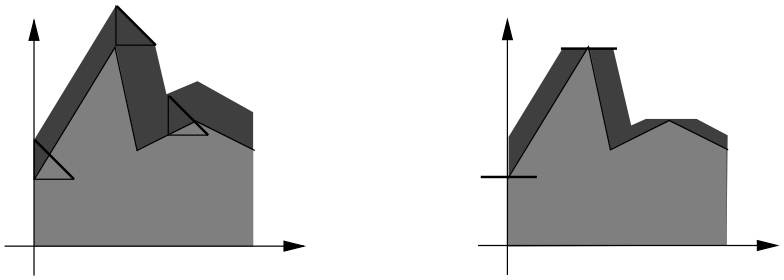


Eléments symétriques
$\tilde{B}(s) = B(-s)$

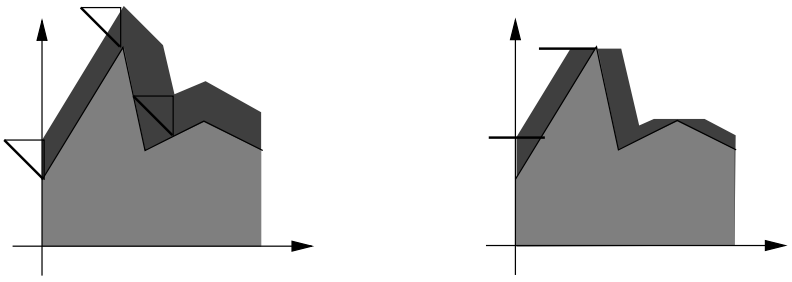


addition de Minkowski :

$A \oplus B = \bigcup_{a+b} a + b$



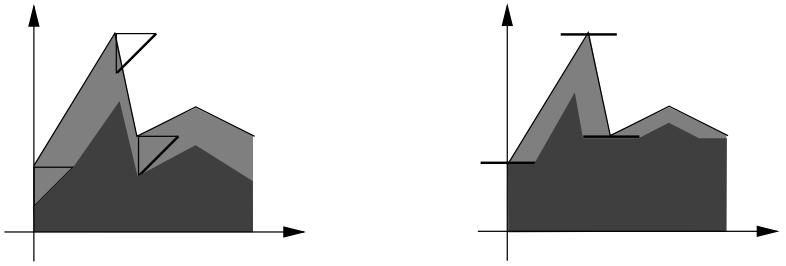
$A \oplus B = \{ (s,z), \; z <= \sup_{t} (A(s-t) + B(t)) \}$



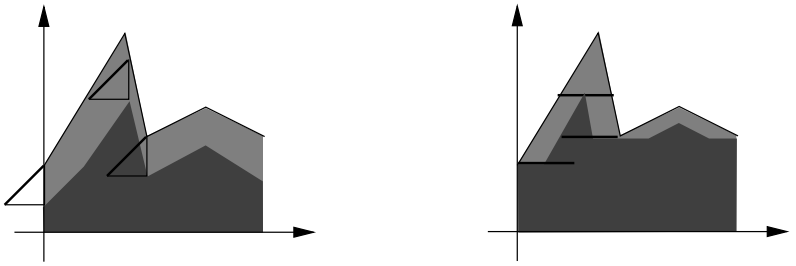
Opérateurs morphologiques sur images en ng

soustraction de Minkowski :

$A \ominus B = (A^c + B^c)$

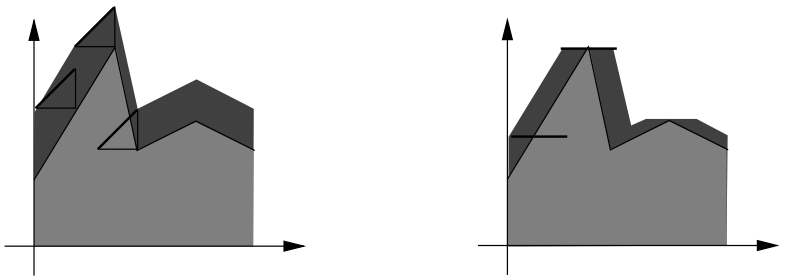


$A \ominus B = \{ (s,z),\ z <= \inf_t (A(s-t) - B(t)) \}$



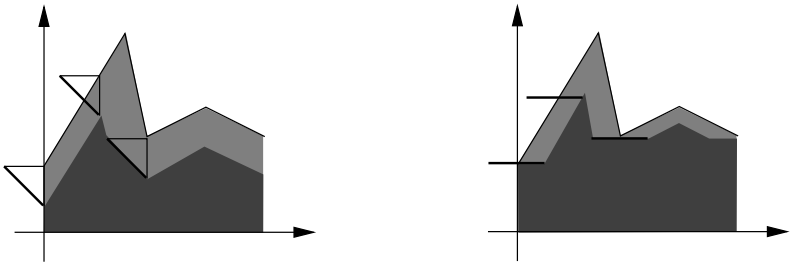
dilatation:

$D(A,B) = A \oplus \tilde{B}$



érosion :

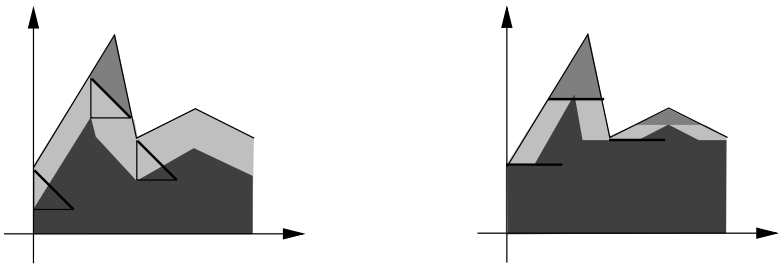
$E(A,B) = A \ominus \tilde{B}$



Opérateurs morphologiques sur images en ng

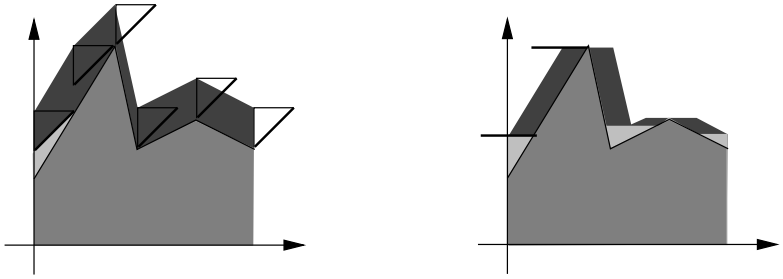
ouverture :

$\Omega_B(A) = (A \ominus \tilde{B}) \oplus B$ $\quad A \ominus \tilde{B}$



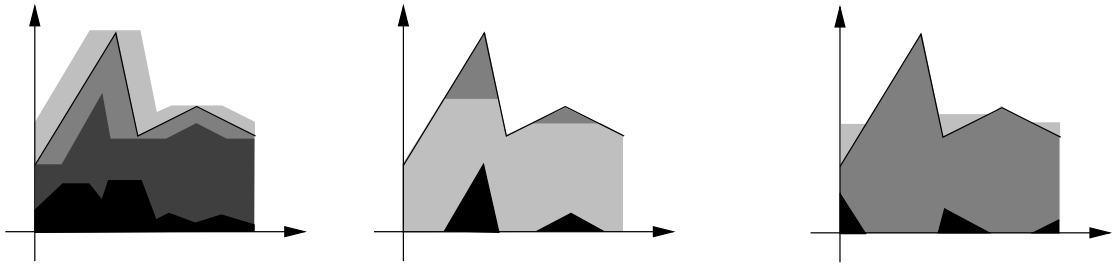
fermeture :

$\Phi_B(A) = (A \oplus \tilde{B}) \ominus B$ $\quad A \oplus \tilde{B}$



gradient : correction de fond par chapeau haut de forme:

$(A \oplus \tilde{B} - A \ominus \tilde{B})/2$ $\quad A - \Omega_B(A)$ $\quad \Phi_B(A) - A$



$A \ominus \tilde{B}$

$A \oplus \tilde{B}$

$\Omega_B(A)$

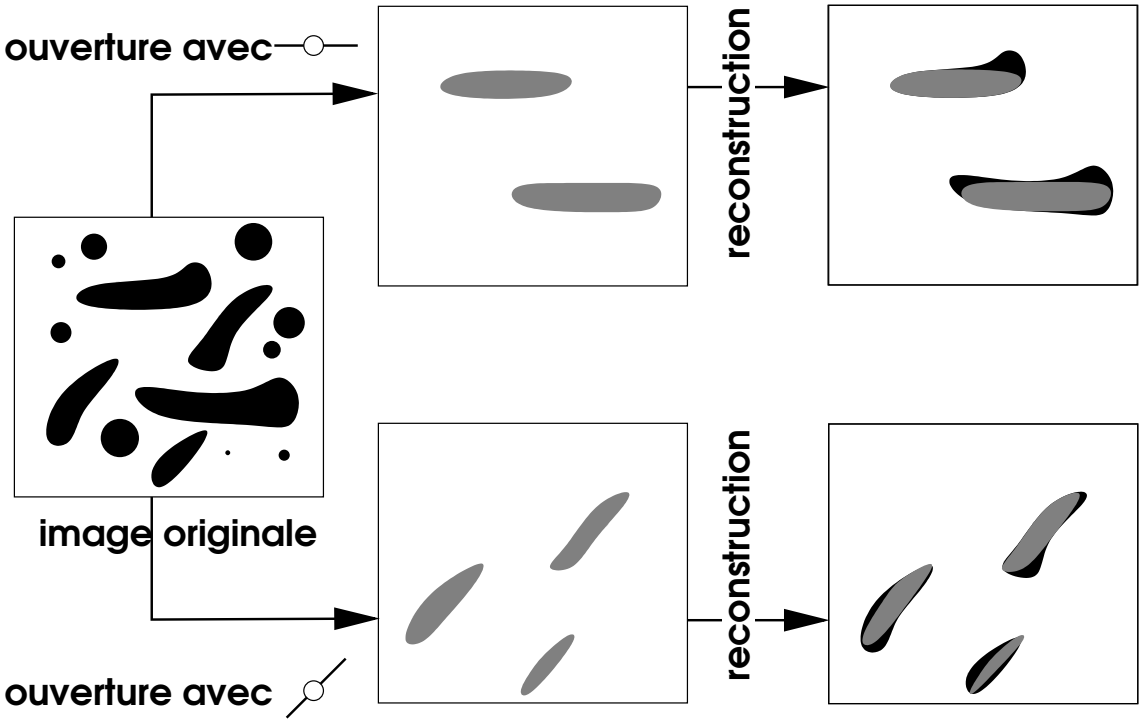
zones claires

$\Phi_B(A)$

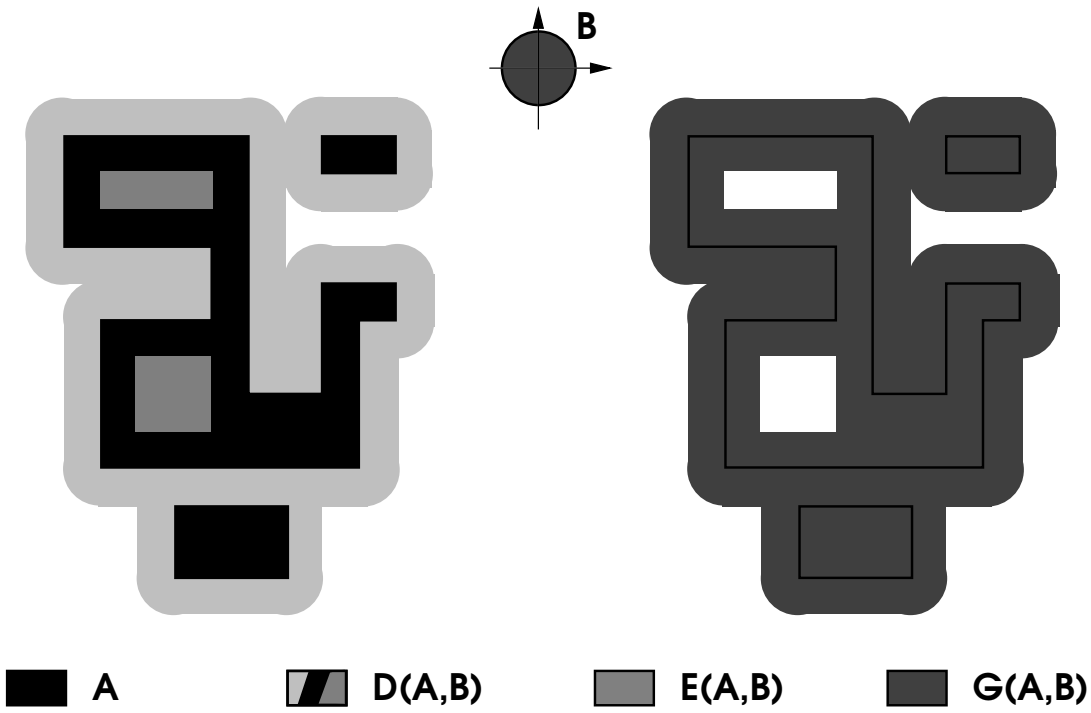
zones sombres

Opérateurs morphologiques sur images en ng

application à la granulométrie :



segmentation de contours par approximation du gradient :

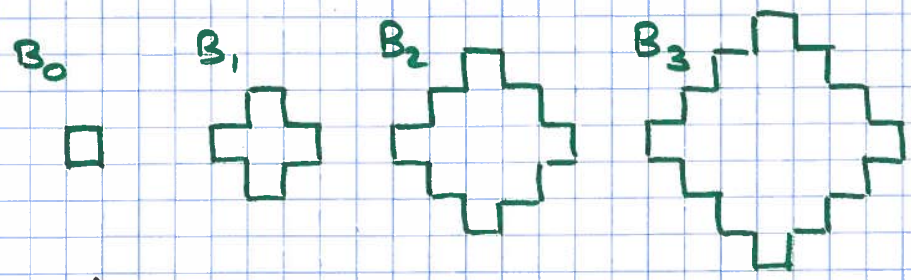


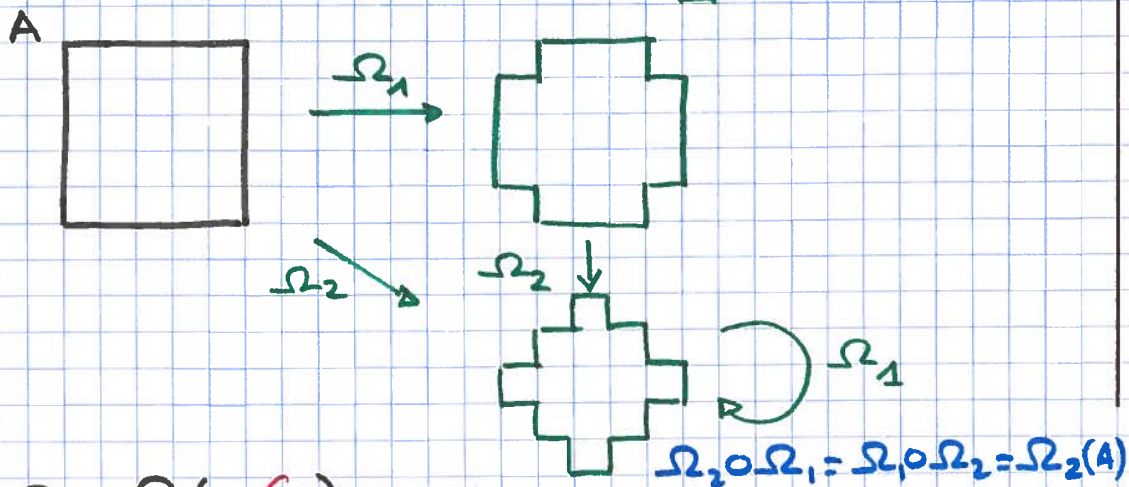
Applications de la morphologie mathématique

GRANULOMÉTRIE

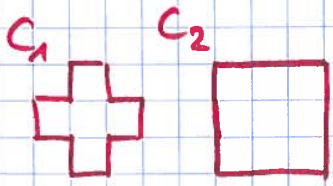
Famille d'ouvertures $(\Omega_i)_i$: $i \leq j \Rightarrow \Omega_i \circ \Omega_j = \Omega_j \circ \Omega_i = \Omega_j$

Ex : $\Omega_i = \Omega(\cdot, B_i)$

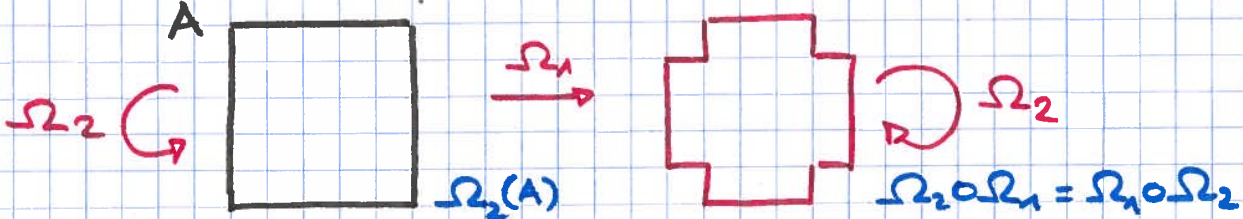




Contre Ex : $\Omega_i = \Omega(\cdot, C_i)$



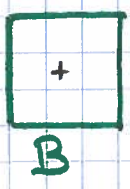
Pourtant $C_1 \subset C_2$

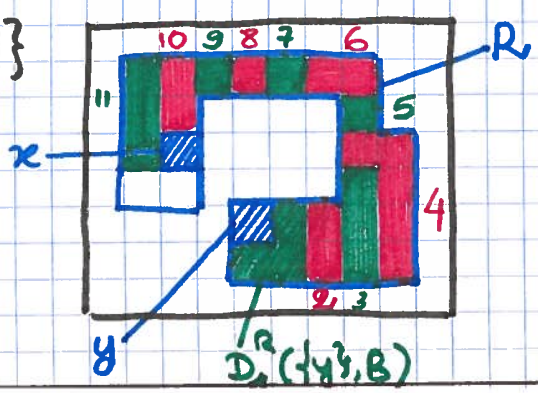


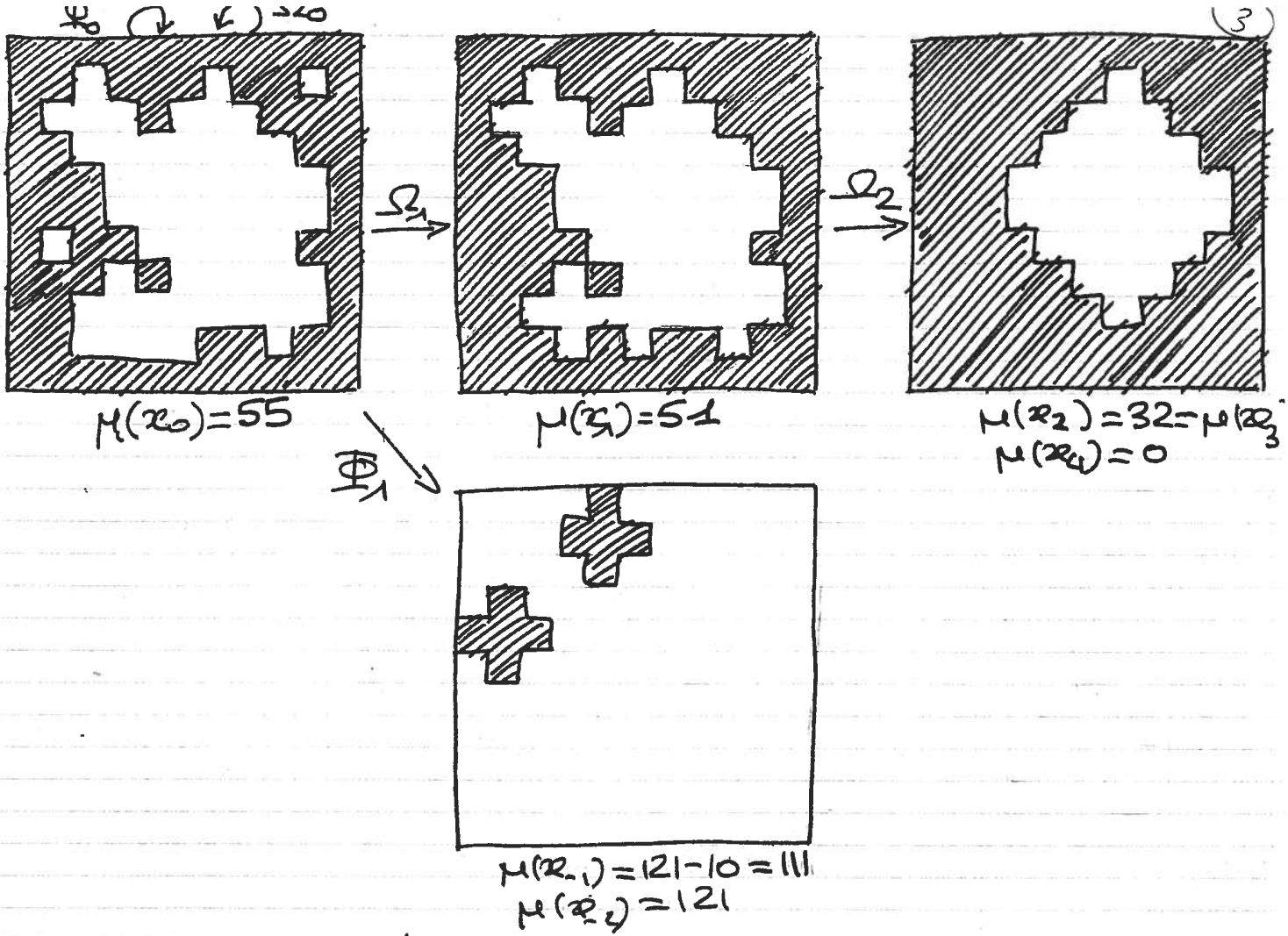
DISTANCE GÉODÉSIQUE

$d_R(x,y) = \min\{ i : x \in D_i^R(\{y\}, B) \}$

$= 11$

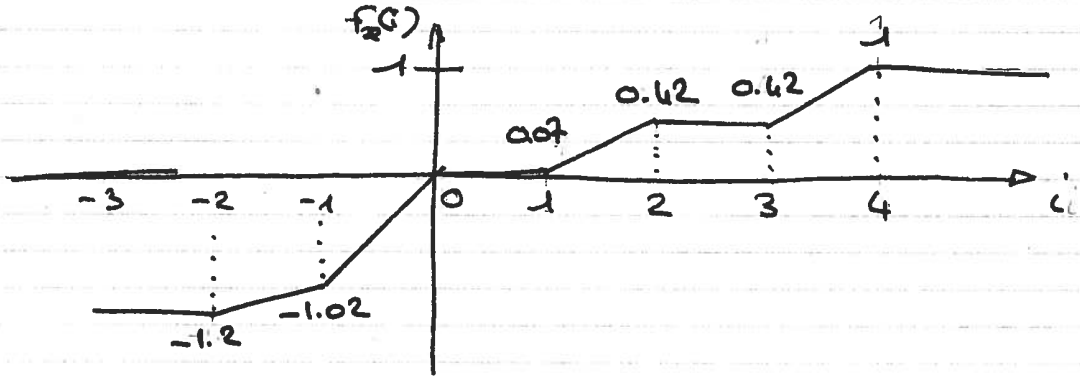




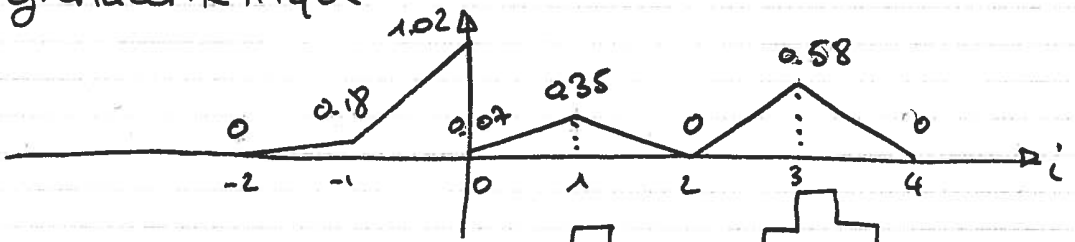


Fonction de distribution granulométrique

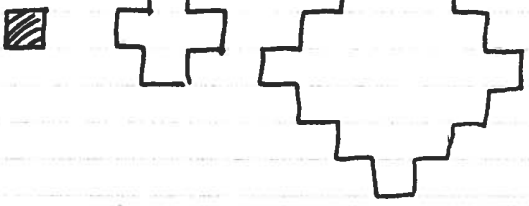
$$F_X(i) = 1 - \frac{\mu(X_i)}{\mu(X_0)} = \frac{55 - \mu(X_i)}{55}$$

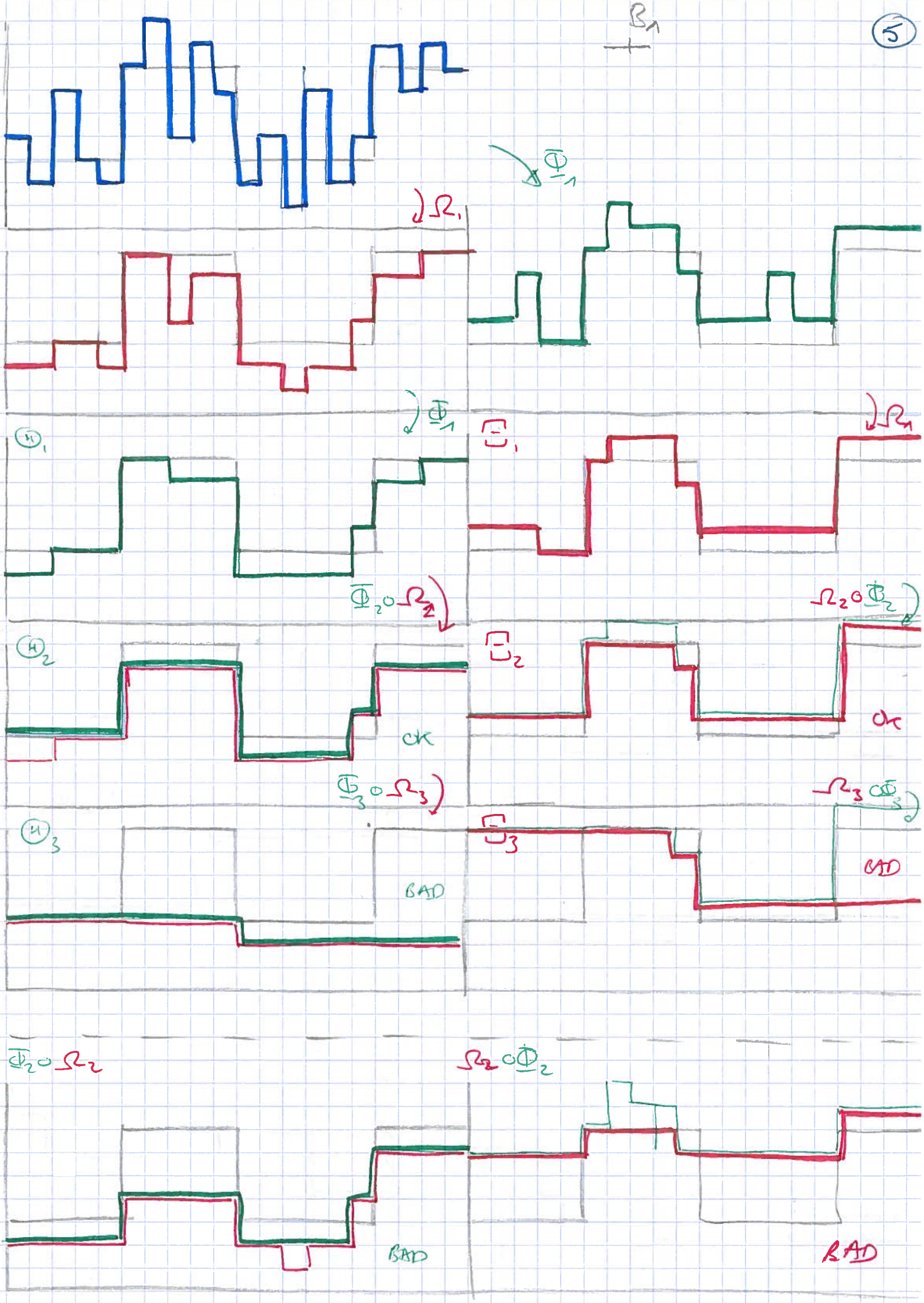


Spectre granulométrique



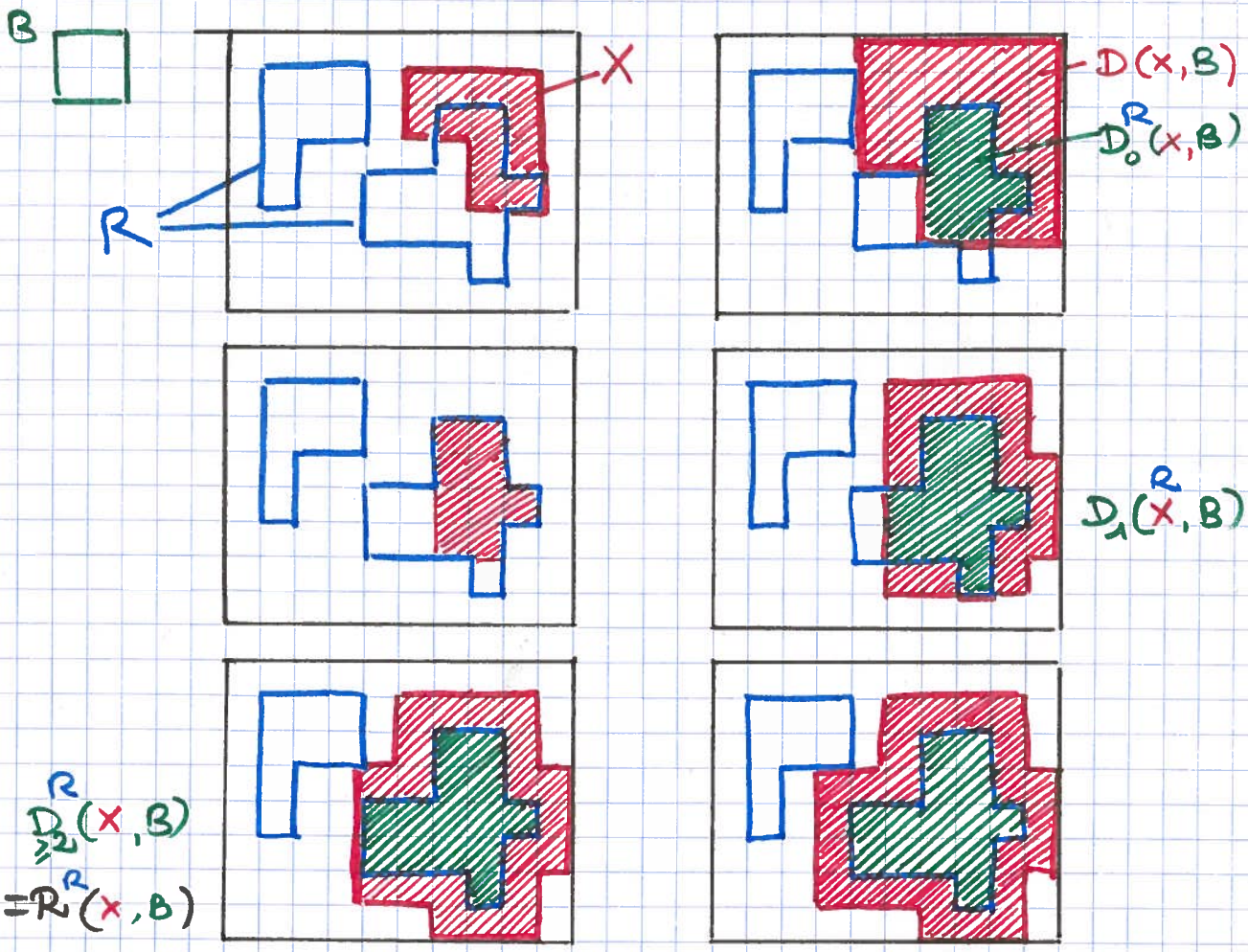
Analyse granulométrique:



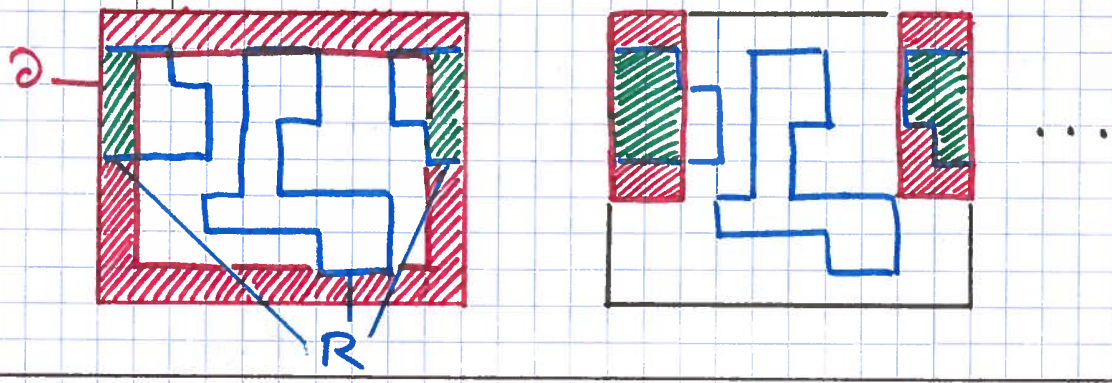

Ω_1
Φ_1
Ω_1
Φ_1
Φ_1
Ω_1
$\Phi_2 \circ \Omega_2$
$\Omega_2 \circ \Phi_2$
OK
OK
$\Phi_3 \circ \Omega_3$
$\Omega_3 \circ \Phi_3$
BAD
BAD
$\Phi_2 \circ \Omega_2$
$\Omega_2 \circ \Phi_2$
BAD
BAD

RECONSTRUCTION GÉODÉSIQUE (1/2)

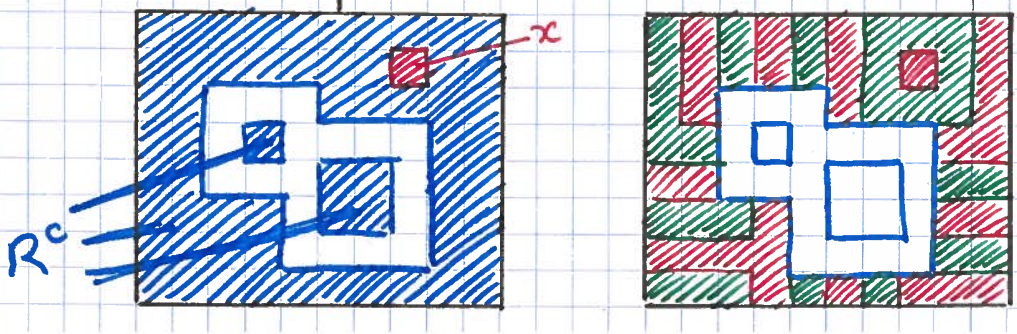
$$\mathcal{R}^R(X,B) = \sup_{i\geq 0} \{ D_i^R(X,B) \}$$



Suppression des objets touchant le bord : $R - \mathcal{R}^R(\partial, B)$

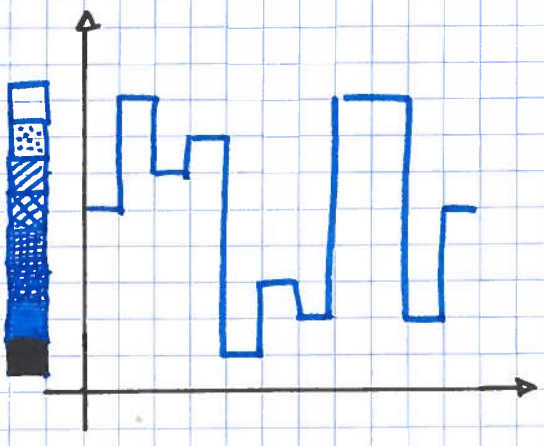


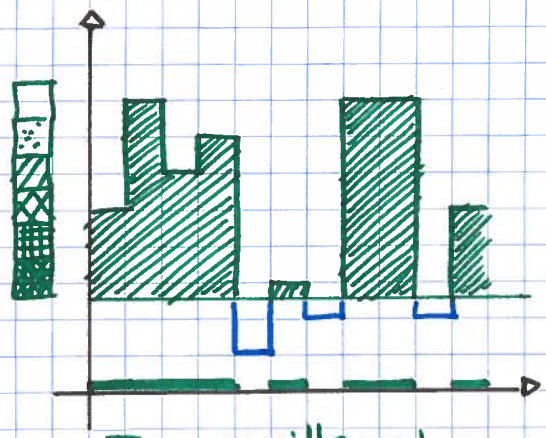
Boucher les trous : complément de $\mathcal{R}^{R^c}(x,B)$ où $x \in R^c$



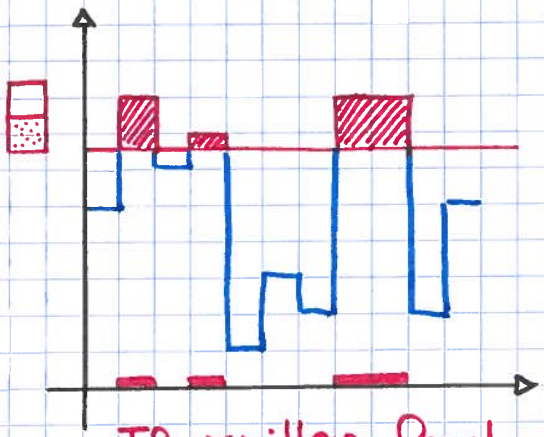
RECONSTRUCTION GÉODÉSIQUE (2/2)

Seuillage par Hystérésis

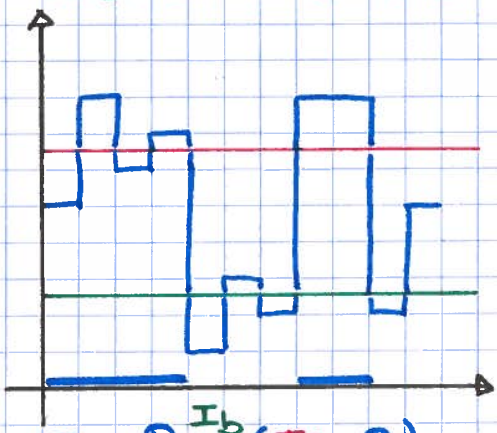




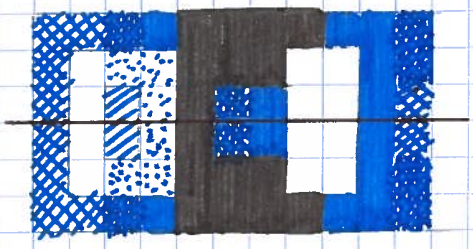
I_b : seuillage bas

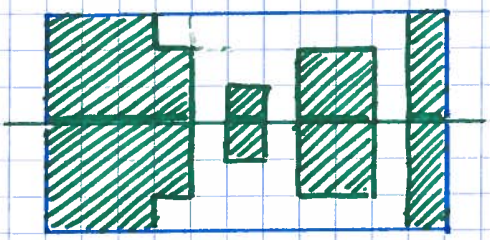


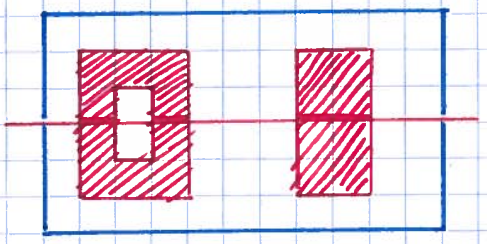
I_h : seuillage haut

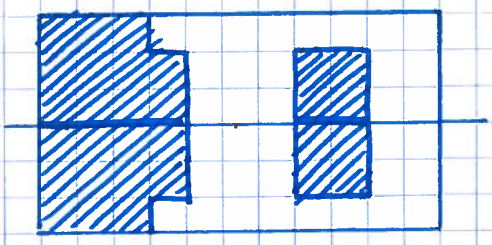


$I_y = R^{I_b}(I_h, B)$



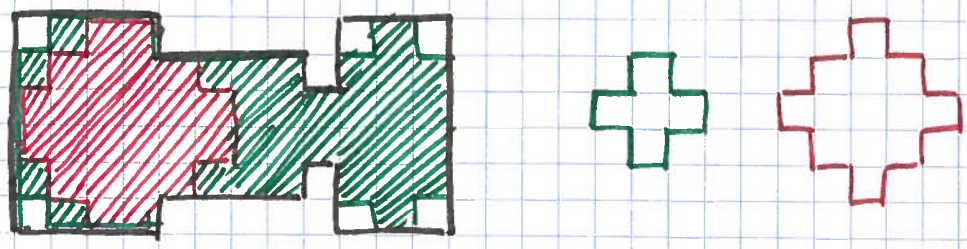




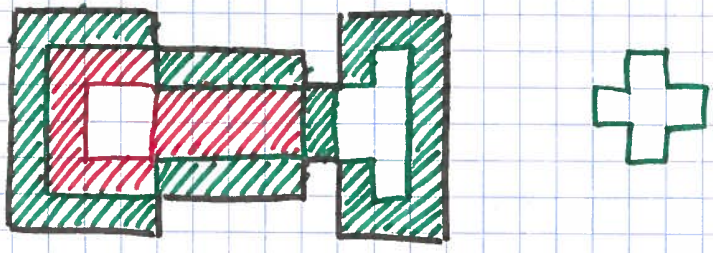


SÉPARATION D'OBJETS

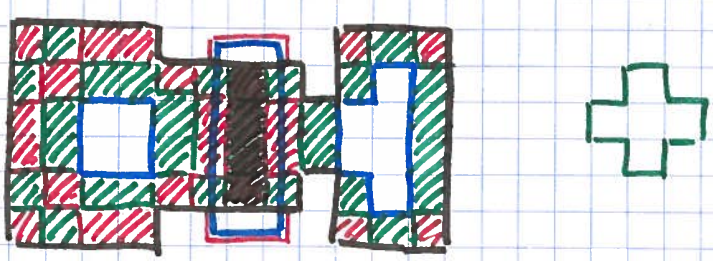
Insuffisance de l'ouverture



Compter les objets : érodés ultimes (⇒ germes)



Skiz géodésique : dilatation géodésique des germes



Ligne de partage des eaux : $\bigcup_i$ skiz de érodé_{i-1} dans érodé_i