Blind extraction of intermittent sources

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Abstract. In this work, we tackle the problem of blind extraction of intermittent sources. Our approach is based on the generalized eigenvector decomposition of covariance matrices and extends previous works in two aspects: by developing a more precise technique to detect inactive periods and by building a more general yet more precise strategy to estimate the vectors that lead to the separation of the intermittent sources. Simulations are carried out to illustrate the effectiveness of our proposal.

Key words: blind source separation, intermittent sources, second-order methods, generalized eigenvector decomposition

1 Introduction

Blind source separation (BSS) concerns the retrieval of a set of signals (sources) by considering only mixed versions of these original sources. When a linear model is assumed, the mixtures $\mathbf{x}(t) = [x_1(t), \cdots, x_N(t)]^T$ are linked to the unknown sources $\mathbf{s}(t) = [s_1(t), \cdots, s_M(t)]^T$ by

$$\mathbf{x}(t) = A\mathbf{s}(t), \tag{1}$$

where $A \in \mathbb{R}^{N\times M}$ is the unknown mixing matrix. When $N \geq M$, independent component analysis (ICA) [9, 3, 2] methods can be employed to perform source separation in (1). In short, ICA, which works under the assumption that the sources are statistically mutually independent, looks for a separating matrix B that makes the retrieved sources $\mathbf{y}(t) = B\mathbf{x}(t)$ as independent as possible. Alternatively, in second-order methods, BSS is accomplished by exploiting the time structure of the sources (coloration [1] or non-stationarity [12]). In these approaches, as well as in ICA, neither the scaling of the sources nor their original order can be identified.

More recent studies in BSS have been suggesting that it is possible to obtain a better performance or even tackle underdetermined cases by considering prior information that are not present in the basic ICA and second-order methods framework. For instance, one can make use of the fact that the sources can be represented, possibly in a transformed domain, by sparse signals [8].

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In this paper, we address the extraction of intermittent sources. This assumption, which is closely related to non-stationarity and to sparsity, is based on the observation that the sources are inactive in some windows, possible in a transformed domain —such an hypothesis holds, for instance, when representing speech signals in the temporal or time-frequency domains [13, 4], or chemical signals in the frequency domain [5, 6]. Source separation of intermittent sources can be carried out by means of a generalized eigenvalue decomposition (GEVD) of two covariance matrices. A first attempt in this direction could be found in [11]. However, this method does not explain how to choose practically these two covariance matrices. To overcome this difficulty one may use a joint diagonalization of several covariance matrices [12]. Nevertheless, there is an overhead in this later method as it separates all the sources and not only the intermittent sources.

More recent studies [13, 6] proposed methods based on the GEVD that are specially tailored for the extraction of intermittent sources. The method introduced in the present work extends [13, 6] in two aspects: 1) by improving the detection of inactivity periods and 2) by exploiting in a better way these inactivity periods (*i.e.* without the need of a deflation procedure as in [6] or the restriction that, to separate a given source, say s_i, there should be a period when only s_i is inactive [13]). This article is organized as follows. Section 2 presents the proposed approach to exploit the inactivity periods of intermittent sources. The proposed algorithm is introduced in Section 3. Numerical experiments and results are given in Section 4 before conclusion and perspectives in Section 5.

2 Basics

In this section, the principles underlying the proposed method to extract intermittent sources are presented. It is based on a second-order framework. Let consider the linear instantaneous mixing model (1) with as many observations as sources ($M = N$). Also, let us represent the covariance matrix of mixtures $\mathbf{x}(t)$ at sample t by

$$R_{\mathbf{x}}(t) \triangleq E[\mathbf{x}(t)\mathbf{x}(t)^T] = \sum_{i=1}^{N} \sigma_i^2(t)\mathbf{a}_i\mathbf{a}_i^T, \tag{2}$$

where $\sigma_i^2(t) = E[s_i(t)^2]$ is the power of the i-th source at sample t and $\mathbf{a}_i$ is the i-th column of mixing matrix $A = [\mathbf{a}_1, \cdots, \mathbf{a}_N]$.

The proposed method is based on the assumption that there exist some samples where at least one source is inactive: *i.e.* for $t = \tau$, $\exists\, n \;/\; s_n(\tau) = 0$. Let suppose in this section that all the sources are stationary excepted N_1 sources, say $s_1(t), \cdots, s_{N_1}(t)$ without loose of generality. Moreover, we assume that the first N_1 sources are simultaneously inactive at sample τ: *i.e.* for $t = \tau$, $1 \leq i \leq N_1,\ s_i(\tau) = 0$. Therefore the covariance matrix of observations $\mathbf{x}(t)$ at sample τ can be written as

$$R_{\mathbf{x}}(\tau) = \sum_{i=N_1+1}^{N} \sigma_i^2\mathbf{a}_i\mathbf{a}_i^T, \tag{3}$$

where for the $N - N_1$ stationary sources $\sigma_i^2 = \sigma_i^2(\tau) = \sigma_i^2(t)$. The proposed method is based on the generalized eigenvalue decomposition of the couple $(R_{\mathbf{x}}(\tau), R_{\mathbf{x}}(t))$. It is easy to show that $(R_{\mathbf{x}}(\tau), R_{\mathbf{x}}(t))$ admits only two distinct generalized eigenvalues: 0 degenerated N_1 times whose eigensubspace $\mathcal{E}_0$ is orthogonal to the space spanned by $\{\mathbf{a}_{N_1+1}, \cdots, \mathbf{a}_N\}$, and 1 degenerated $N - N_1$ times whose eigensubspace $\mathcal{E}_1$ is complementary to $\mathcal{E}_0$ in $\mathbb{R}^N$. As a consequence, the projection of the observations $\mathbf{x}(t)$ onto $\mathcal{E}_0$ can be used to cancel the contribution of the sources $s_i(t)$, $N_1 + 1 \leq i \leq N$. This means that all separation vectors $\mathbf{b}_i$, $1 \leq i \leq N_1$, lie in $\mathcal{E}_0$, where $\mathbf{b}_i$ is the i-th column of the separation matrix B. In other words, the space spanned by $\{\mathbf{b}_1, \cdots, \mathbf{b}_{N_1}\}$ is $\mathcal{E}_0$. It is worth noting that when only one source is inactive (let say $s_i(t)$), then $\mathcal{E}_0$ is unidimensional and the corresponding generalized eigenvector is aligned with $\mathbf{b}_i$.

This method allows us firstly to detect how many sources are vanishing by testing the generalized eigenvalues and then to extract the space spanned by the corresponding sources by projecting the observations onto the generalized eigenvectors associated with the generalized eigenvalues equal to zero.

3 Algorithm to extract intermittent sources

The previous section discussed how to extract the space spanned by intermittent sources but not how to extract these sources. Moreover, the inactivity periods are unknown. Given that, we propose, in a first part of this section, a strategy to detect inactivity periods that extends the one proposed in [13]. Then, we show how to estimate the extraction vectors $\mathbf{b}_i$ based on the subspaces observed during the inactive periods.

3.1 Estimation of inactivity periods

In order to detect block samples where at least one source is inactive, we proposed to compute for different samples τ the generalized eigenvalue decomposition of couples $\{(R_{\mathbf{x}}(\tau), R_{\mathbf{x}})\}_\tau$, where $R_{\mathbf{x}}$ is the covariance matrix of observations $\mathbf{x}(t)$ estimated with all samples, and $R_{\mathbf{x}}(\tau)$ is the covariance matrix of the observations estimated on windowed samples around τ. The generalized eigendecomposition of $(R_{\mathbf{x}}(\tau), R_{\mathbf{x}})$ provides

$$R_{\mathbf{x}}(\tau)V(\tau) = R_{\mathbf{x}}V(\tau)\Lambda(\tau), \tag{4}$$

where $\Lambda(\tau)$ is a diagonal matrix whose diagonal entries $\lambda_1(\tau) \leq \cdots \leq \lambda_N(\tau)$ are the generalized eigenvalues and $V(\tau)$ is an orthonormal matrix whose columns $\mathbf{v}_i(\tau)$ are the generalized eigenvectors. Let us define g_k such that

$$g_k(\tau) = \prod_{i=1}^{k} f_0(\lambda_i(\tau)) \prod_{j=k+1}^{N} f_1(\lambda_j(\tau)), \tag{5}$$

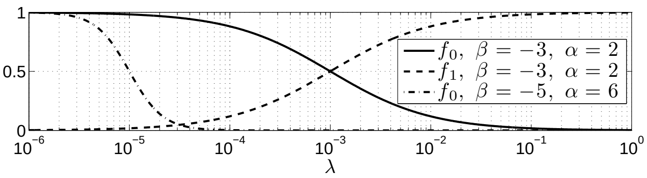


Fig. 1. Detection of inactive sources.

where

$$f_0(\lambda) = 1 - \frac{1}{1 + \exp(-\alpha(\log(\lambda) - \beta))} \quad \text{and} \quad f_1(\lambda) = \frac{1}{1 + \exp(-\alpha(\log(\lambda) - \beta))}.$$

Note that these functions are sigmoids (Fig. 1): if $\log(\lambda) - \beta$ is large compared to $1/\alpha$ then $f_0(\lambda)$ (resp. $f_1(\lambda)$) is about 0 (resp. 1). Accordingly,

$$\hat{k}(\tau) = \arg_k g_k(\tau) > .5$$

is an estimation of the number of inactive sources at sample τ (*i.e.* less powerful than 10^β times their average power), and the subspace $\mathcal{E}_0(\tau, \hat{k}(\tau))$ spanned by $\{\mathbf{v}_1(\tau), \cdots, \mathbf{v}_{\hat{k}(\tau)}(\tau)\}$ is also spanned by $\hat{k}(\tau)$ of the separation vectors $\mathbf{b}_i$. Finally, the identification of inactivity periods $\Theta = \{\tau \,|\, g_k(\tau) > .5\}$ also provides couples of weight and subspace[1] $\left\{(g_{\hat{k}(\tau)}(\tau), \mathcal{E}_0(\tau, \hat{k}(\tau))\right\}$. We shall discuss later how to make use of these weights to improve the estimation of the extraction vectors $\mathbf{b}_i$.

3.2 Estimation of extraction vectors

Once the inactivity periods Θ are detected by the previous step, the set of subspaces $\{\mathcal{E}_0(\tau, \hat{k}(\tau))\}_{\tau \in \Theta}$ is used to estimate the separation vectors $\mathbf{b}_i$. It is worth noting that when $\hat{k}(\tau) = 1$ then the corresponding $\mathbf{v}_1(\tau)$ is directly align with one of the separation vectors $\mathbf{b}_i$. However, when $\hat{k}(\tau) \neq 1$, then k separation vectors $\mathbf{b}_i$ lie in the k-dimensional subspace $\mathcal{E}_0(\tau, \hat{k}(\tau))$ but are not necessary align with the eigenvectors $\mathbf{v}_i(\tau)$. To overcome this difficulty, our proposal considers the method for finding the intersection of subspaces described in [7] (see also Appendix). It is interesting to observe that, if $\dim(\mathcal{E}_0(\tau, k) \cap \mathcal{E}_0(\tau', k')) = 1$, then the support vector $\mathbf{u}_{\tau,\tau'}$ lying in the intersection between these two subspaces must be aligned with one of the separation vector $\mathbf{b}_i$. Accordingly searching all the intersections between $\mathcal{E}_0(\tau, \hat{k}(\tau))$ and $\mathcal{E}_0(\tau', \hat{k}(\tau'))$, with $(\tau, \tau') \in \Theta^2$, such that $\dim(\mathcal{E}_0(\tau, \hat{k}(\tau)) \cap \mathcal{E}_0(\tau', \hat{k}(\tau'))) = 1$ provides a set of support vectors

$$\mathcal{U} = \left\{\mathbf{u}_{\tau,\tau'} \mid \dim\Big(\mathcal{E}_0(\tau, \hat{k}(\tau)) \bigcap \mathcal{E}_0(\tau', \hat{k}(\tau'))\Big) = 1\right\}. \tag{6}$$

[1] In this article, by sake of simplicity we do not make the difference between a subspace and its matrix representation.

Note, however, that more support vectors than separation vectors might be identified. Therefore, in an ideal situation, some separation vectors are repeated. Of course, when there is some noise in the observed data or when the detection of inactivity periods is not completely perfect, which is usually the case, it is expected that these repeated vectors be actually concentrated around the optimum direction. Therefore, our method is completed by a clustering stage whose goal is exactly to estimate the separation vectors $\mathbf{b}_i$ used in the extraction of the intermittent sources.

In this study, we consider kernel-PCA [10] for performing the clustering. The chosen kernel is

$$\psi(\mathbf{u}_{\tau_1,\tau_2}, \mathbf{u}_{\tau_3,\tau_4}) = \begin{cases} \dfrac{|\mathbf{u}_{\tau_1,\tau_2}^T \mathbf{u}_{\tau_3,\tau_4}| - \cos(\theta_0)}{1 - \cos(\theta_0)}, & \text{if } |\mathbf{u}_{\tau_1,\tau_2}^T \mathbf{u}_{\tau_3,\tau_4}| \geq \cos(\theta_0) \\ 0, & \text{else} \end{cases} \tag{7}$$

where $\mathbf{u}_{\tau_i,\tau_j} \in \mathcal{U}$. θ_0 is an *a priori* chosen angle which defines the minimum angle between two separation vectors. In practice, the accuracy of support vector $\mathbf{u}_{\tau_i,\tau_j}$ depends on the values of the corresponding eigenvalues $\lambda_l(\tau_i)$ and $\lambda_l(\tau_j)$. It can be shown from performance analysis that the accuracy of $\mathcal{E}_0(\tau_i, \hat{k}(\tau_i))$ increases as the related eigenvalues $\lambda_1(\tau_i), \cdots, \lambda_{\hat{k}(\tau_i)}(\tau_i)$ decrease to zero. In view of the observation of the last paragraph, we propose the following weighted version of the kernel (7)

$$\psi'(\mathbf{u}_{\tau_1,\tau_2}, \mathbf{u}_{\tau_3,\tau_4}) = w_{\tau_1,\tau_2}\, w_{\tau_3,\tau_4}\, \psi(\mathbf{u}_{\tau_1,\tau_2}, \mathbf{u}_{\tau_3,\tau_4}), \tag{8}$$

where w_{τ_i,τ_j} is a measure of the inactivity accuracy defined by

$$w_{\tau_i,\tau_j} = \sqrt{g_{k_i}(\tau_i) g_{k_j}(\tau_j)}. \tag{9}$$

Note here that, by proceeding this way, the support vectors $\mathbf{u}_{\tau_i,\tau_j}$ that are associated with lower eigenvalues through $g_{\hat{k}(\tau)}$ are somehow more important in the clustering step. The kernel PCA consists in performing an eigenvalue decomposition of matrix $\Psi' \in \mathbb{R}^{\mathrm{card}(\mathcal{U})\times \mathrm{card}(\mathcal{U})}$ whose entries are $\psi'(\mathbf{u}_{\tau_1,\tau_2}, \mathbf{u}_{\tau_3,\tau_4})$:

$$\Psi' = \Phi \Delta \Phi^T, \tag{10}$$

where Δ is a diagonal matrix of eigenvalues and Φ is an orthonormal matrix whose columns are eigenvectors of Ψ'. Let $W = [\boldsymbol{\phi}_1, \cdots, \boldsymbol{\phi}_n]$ be the matrix of the n eigenvectors related to the n largest eigenvalues. The extraction matrix $B \in \mathbb{R}^{n\times N}$ is then estimated by

$$B = W^T \Psi' U^T, \tag{11}$$

where U is the matrix obtained by the concatenation of all support vectors in $\mathcal{U}$. Note that the exact number n of intermittent sources has not to be known *a priori* since it can be estimated by checking the eigenvalues $\Delta_{i,i}$ (10). The intermittent sources are finally estimated thanks to

$$\hat{\mathbf{s}}(t) = B\mathbf{x}(t), \tag{12}$$

for all samples t, including those when the intermittent sources are active. The final algorithm is summarized in Algo. 1.

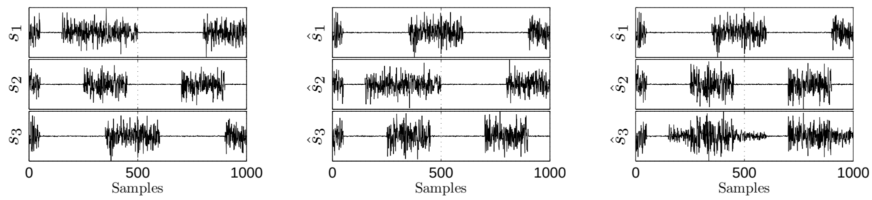
Algorithm 1 Proposed algorithm

{Estimation of the inactivity periods}
  Compute covariance matrix $R_{\mathbf{x}}$ from all samples
  for each τ **do**
    Compute covariance matrix $R_{\mathbf{x}}(\tau)$ from sliding windows centered on τ
    Compute generalized eigenvalue decomposition (4) and for $1 \leq k \leq N$, $g_k(\tau)$ (5)
    Estimate the number $\hat{k}(\tau)$ of inactive sources thanks to $\hat{k}(\tau) = \arg_k g_k(\tau) > .5$
    $\Rightarrow \left(g_{\hat{k}(\tau)}(\tau), \mathcal{E}_0(\tau, \hat{k}(\tau)\right)$
  end for
  Estimate the set of inactivity periods by $\Theta = \{\tau \mid \hat{k}(\tau) \geq 1\}$

{Estimation of the subspaces intersections}
  for each $1 \leq i, j \leq \text{card}(\Theta)$ **do**
    Dimension of the intersection: $d(i,j) = \dim(\mathcal{E}_0(\tau_i, \hat{k}(\tau_i)) \cap \mathcal{E}_0(\tau_j, \hat{k}(\tau_j)))$
  end for
  Estimate the set of support vectors $\mathcal{U}$ (6) such that $d(i,j) = 1$

{Estimation of extraction vectors}
  Compute weighted kernel PCA of $\{(\mathbf{u}_{i,j}, w_{i,j})\}$ (7) to (11)

  Estimate intermittent sources by $\hat{\mathbf{s}}(t) = B\mathbf{x}(t)$ (12)



(a) Actual sources (b) Estimated sources (c) Estimated sources [13]

Fig. 2. Illustration of the proposed methodology.

4 Numerical experiments

The first experiment illustrates the influence of the new methodology to estimate extraction vectors (Fig. 2). Three artificial intermittent sources (Fig. 2(a)) are linearly mixed. As one can see, the three sources are chosen so that the first one is never the only inactive source. In this case, our previous method [13] (Fig. 2(c)) has failed to extract the first actual source: the third estimated source is still a mixture of several actual sources. On the contrary, the proposed methodology (Fig. 2(b)) has succeeded to extract all the three sources without deflation.

The second experiment compares the extraction of 5 speech signals from 70 mixtures of audio signals by the proposed algorithm and a more classical estimation of the separation matrix by joint-diagonalization of covariance matrices based on non stationarity [12] (refered as SONS). In this experiment, the results are averaged over 50 randomly chosen configurations: the 5 speech signals, the

	Proposed method	SONS [12]
PI_{dB} (13)	-34	-41
Time [s]	1.1	847

Table 1. Comparison of speech sources extraction.

65 audio (musical) signals and the mixing matrix are randomly chosen. All the signals are sampled at 16kHz. The time sliding windows have a 40ms length with an overlap of 50%. To evaluate the estimation of the extraction vectors $\mathbf{b}_i$, we have used the performance index defined by

$$PI_{dB} = 10 \log \left(\frac{1}{\mathrm{card}(\mathcal{S})} \sum_{i \in \mathcal{S}} \left(\sum_j \left| \frac{C_{i,j}}{\max_k |C_{i,k}|} \right|^2 - 1 \right) \right), \text{ with } C = B\,A, \quad (13)$$

where $\mathcal{S}$ denotes the set of speech sources. So the smaller the performance index is, the better the extraction is. As one can see (Tab 1), the proposed method is slightly less performant than the SONS method while keeping quite good performance. Indeed, it only exploits a part of the signal while the SONS used the overall signals. However, it is worth noting that the proposed method has a less computational coast.

5 Conclusions and perspectives

In this paper, we introduced an algorithm to extract intermittent sources from linear mixtures. It is based on second order statistics: the detection of inactivity periods allows to estimate the separation matrix which is used to extract the intermittent sources when they are active. Simulations in different configurations pointed out that our proposal is efficient and presents a low computational cost. Even if in this study the purpose was to extract speech signals, the proposed algorithm can be used in a more general context. Future works include the derivation of an automatic strategy for adjusting parameters. Moreover, a more robust clustering algorithm will also be studied. Finally, this proposed method could be used as an initialization of joint diagonalization of covariance matrices to only extract intermittent sources.

Appendix: intersection of subspaces

This appendix summarizes some useful considerations about intersection of subspaces [7]. Let $\mathcal{S}_1$ and $\mathcal{S}_2$ be two subspaces in $\mathbb{R}^m$. The principal angles θ_k between subspaces $\mathcal{S}_1$ and $\mathcal{S}_2$ are defined by $\cos(\theta_k) = \max_{\mathbf{u} \in \mathcal{S}_1} \max_{\mathbf{v} \in \mathcal{S}_2} \mathbf{u}^T \mathbf{v} = \mathbf{u}_k^T \mathbf{v}_k$,subject to $\|\mathbf{u}\|_2 = \|\mathbf{v}\|_2 = 1$, $\mathbf{u}^T \mathbf{u}_i = 0$, and $\mathbf{v}^T \mathbf{v}_i = 0$, $\forall\, i = \{1, \cdots, k-1\}$. Let $Q_1 \in \mathbf{R}^{m \times p}$ and $Q_2 \in \mathbf{R}^{m \times q}$ be two orthonormal basis of $\mathcal{S}_1$ and $\mathcal{S}_2$, respectively. Assume that $p \geq q$. The angles between subspaces can be efficiently obtained by the singular value decomposition of $Q_1^T Q_2$:

$$Y^T \big(Q_1^T Q_2\big) Z = \mathrm{diag}(\lambda_1, \cdots, \lambda_q).$$

The principal vectors and angles are then obtained by $[\mathbf{u}_1, \cdots, \mathbf{u}_p] = Q_1 Y$, $[\mathbf{v}_1, \cdots, \mathbf{v}_q] = Q_2 Z$ and $\cos(\theta_k) = \lambda_k,\ k = \{1, \cdots, q\}$, respectively. As a consequence, if there exist some principal angles such that $\cos(\theta_k) = 1$, then they define the intersection between subspaces $\mathcal{S}_1$ and $\mathcal{S}_2$. The dimension of the intersection is thus defined by the number of principal angles such that $\cos(\theta_k) = 1$. Also, the intersection is spanned by $\{\mathbf{v}_k\}_k$ or by $\{\mathbf{u}_k\}_k$.

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