THEORETICAL ANALYSIS OF XDAWN ALGORITHM: APPLICATION TO AN EFFICIENT SENSOR SELECTION IN A P300 BCI

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ABSTRACT

A Brain-Computer Interface (BCI) is a specific type of human-machine interface that enables communication between a subject/patient and a computer by a direct control from the decoding of brain activity. To improve the ergonomics and to minimize the cost of such a BCI, reducing the number of electrodes is mandatory. A theoretical analysis of the subjacent model induced by the BCI paradigm leads to derive a closed form theoretical expression of the spatial filters which maximize the signal to signal-plus-noise ratio. Moreover, this new formulation is useful to improve a previously introduced method to automatically select relevant sensors. Experimental results on 20 subjects show that the proposed method is efficient to select the most relevant sensors: from 32 down to 8 sensors, the loss in classification accuracy is less than 2%. Furethermore, the computational time required to rank the 32 sensors is reduced by a 4.6 speed up factor allowing dynamical monitoring of sensor relevance as a marker of the user's mental state.

1. INTRODUCTION

Brain-Computer Interfaces (BCI) are devices which enable direct communication between the user's brain and a computer without requiring any movement, e.g., by recording electroencephalography (EEG) activity [13]. The BCI problem addressed in this paper concerns the P300-speller, which enables people to write text on a computer [3]. It is based on the oddball paradigm: the task is to discriminate between epochs containing a P300 potential evoked by a (rare) target stimulus and epochs associated with the (frequent) non-target stimuli. This well-known paradigm to explore the brain activity in cognitive science [8] allows us to propose an efficient model of the recorded signals. These latter signals are thus composed of two typical patterns plus an additive noise which models the ongoing brain activity as well as all the artifacts (e.g., muscular, ocular). The first pattern is evoked by the target stimuli and the second one is evoked by all the stimuli (target and non-target ones). As already shown [10, 9], this model is useful to provide an accurate estimation of spatial filters to enhance target evoked potentials by maximizing the signal to signal-plus-noise ratio (SSNR). However, these studies lack an analysis to express the estimated spatial filters theoretically.

Moreover, the reduction of the number of EEG sensors is of a great interest to improve the real-life BCI ergonomics. Indeed, using a large number of electrodes is undesirable since it increases the discomfort of the patient, the time needed to install the whole system, its global cost as well as the power consumption for wireless EEG caps and nomad systems. However, only a few studies have focused on sensor selection so far (e.g., [6, 11, 5, 12]). In a previous study [2, 1], we proposed a more efficient sensor selection procedure than classical ones by considering signal to signal-plus-noise ratio (SSNR) instead of the classical classification accuracy.

In this study, a theoretical analysis of the model of the evoked potential paradigm is derived in the case of a P300 speller BCI to give the theoretical expression of the estimated spatial filters. Moreover, this analysis allows us to implement the sensor selection procedure in an elegant and more efficient way than in previous studies.

2. THEORETICAL ANALYSIS OF XDAWN ALGORITHM

In this section, the principles of the xDAWN algorithm are briefly reprised [10, 9] and a new theoretical analysis of it is derived. This method relies on two assumptions:

- i) signal is composed of two typical patterns, one evoked by the target stimuli and one evoked by all stimuli (target and non-target ones);
- ii) patterns evoked by the target stimuli could be enhanced by spatial filtering.

Let $X \in \mathbb{R}^{N_t \times N_s}$ denote the recorded signals, where N_t and N_s are the number of samples and sensors, respectively. The patterns synchronized with the target and non-target stimuli are denoted by $P_1 \in \mathbb{R}^{N_1 \times N_s}$ and $P_2 \in \mathbb{R}^{N_2 \times N_s}$, respectively. As a consequence, the first assumption allows to write the recorded signals X as

$$X = D_1 P_1 + D_2 P_2 + N, \tag{1}$$

where $D_1 \in \mathbb{R}^{N_t \times N_1}$ and $D_2 \in \mathbb{R}^{N_t \times N_2}$ are Toeplitz matrices whose first column entries are set to zero except for those that correspond to target stimuli time indexes

and all stimuli time indexes, respectively. N_1 and N_2 indicate the number of time samples in responses P_1 and P_2. $N \in \mathbb{R}^{N_t \times N_s}$ denotes the residual noise. Under the assumption that the target pattern P_1 is monodimensional, P_1 can be factorized as $P_1 = \mathbf{a}_1 \mathbf{w}_1^T$ where $\mathbf{a}_1 \in \mathbb{R}^{N_1}$ is the temporal pattern and $\mathbf{w}_1 \in \mathbb{R}^{N_s}$ is its spatial distribution over the sensors.

The aim of the xDAWN algorithm is to estimate a spatial filter $\mathbf{u}_1^* \in \mathbb{R}^{N_s}$ so that the SSNR after the spatial filtering is maximized

$$\mathbf{u}_1^* = \arg\max_{\mathbf{u}} \rho(\mathbf{u}) \tag{2}$$

where the SSNR is defined by

$$\rho(\mathbf{u}) = \frac{\mathbf{u}^T \Sigma_1 \mathbf{u}}{\mathbf{u}^T \Sigma_X \mathbf{u}}, \tag{3}$$

with $\Sigma_1 = E[P_1^T D_1^T D_1 P_1]$ and $\Sigma_X = E[X^T X]$. From model (1) and under the assumption that pattern P_1 and concurrent signals $H = D_2 P_2 + N$ are not correlated, SSNR $\rho(\mathbf{u})$ can be rewritten as

$$\rho(\mathbf{u}) = \frac{\sigma_1^2 \left(\mathbf{u}^T \mathbf{w}_1\right)^2}{\mathbf{u}^T \left(\sigma_1^2 \mathbf{w}_1 \mathbf{w}_1^T + R_H\right) \mathbf{u}},$$

with $R_H = E[H^T H]$ and $\sigma_1^2 = E[\mathbf{a}_1^T D_1^T D_1 \mathbf{a}_1]$. In a practical way, since covariance matrices Σ_1 and Σ_X are unknown, xDAWN algorithm maximizes the estimated SSNR

$$\hat{\mathbf{u}}_1 = \arg\max_{\mathbf{u}} \hat{\rho}(\mathbf{u}) \tag{4}$$

where $\hat{\rho}(\mathbf{u})$ is defined by

$$\hat{\rho}(\mathbf{u}) = \frac{\mathbf{u}^T \hat{\Sigma}_1 \mathbf{u}}{\mathbf{u}^T \hat{\Sigma}_X \mathbf{u}}, \tag{5}$$

with $\hat{\Sigma}_1 = (\hat{P}_1^T D_1^T D_1 \hat{P}_1)/N_t$, $\hat{\Sigma}_X = (X^T X)/N_t$. Note that $\hat{P}_1$ is the least mean square estimation of the unknown target evoked response P_1. Since $D_1 P_1$ and $D_2 P_2$ could overlap, $\hat{P}_1$ is estimated from

$$\begin{pmatrix} \hat{P}_1 \\ \hat{P}_2 \end{pmatrix} = \left(D^T D\right)^{-1} D^T X,$$

with $D = [D_1, D_2]$, $\hat{P}_1$ is thus estimated by $\hat{P}_1 = B_1^T X$.

From the definition of estimated SSNR (5), the spatial filter is given by $\hat{\mathbf{u}}_1 = R_X^{-1} \hat{\mathbf{v}}_1$ where

$$\hat{\mathbf{v}}_1 = \arg\max_{\mathbf{v}_1} \frac{\mathbf{v}_1^T Q_X^T B_1 R_1^T R_1 B_1^T Q_X \mathbf{v}_1}{\mathbf{v}_1^T \mathbf{v}_1}$$

and $X = Q_X R_X$ and $D_1 = Q_1 R_1$ are the QR [4] factorizations of X and D_1, respectively. $\hat{\mathbf{v}}_1$ is obtained from the singular value decomposition (SVD) [4] of $R_1 B_1^T Q_X = \Phi \Lambda \Psi^T$ as the right singular vector $\boldsymbol{\psi}_1$ corresponding to the largest singular value. As a consequence,

$$\hat{\mathbf{u}}_1 = R_X^{-1} \boldsymbol{\psi}_1. \tag{6}$$

Moreover, one can rewrite $\hat{P}_1 = R_1^{-1} \Phi \Lambda \Psi^T R_X$. Since $\hat{P}_1 = B_1^T X = B_1^T D_1 P_1 + B_1^T H$, $\mathbf{w}_1$ is thus estimated as

$$\hat{\mathbf{w}}_1 = R_X^T \boldsymbol{\psi}_1. \tag{7}$$

The enhanced signal is finally given by

$$\hat{S}_1 \stackrel{\triangle}{=} X \hat{\mathbf{u}}_1 = D_1 A_1 \hat{\mathbf{u}}_1 + D_2 A_2 \hat{\mathbf{u}}_1 + N \hat{\mathbf{u}}_1. \tag{8}$$

In a practical way, since the two QR factorizations and the SVD can be computationally time consuming, the spatial filter $\hat{\mathbf{u}}_1$ can be efficiently estimated by the generalized eigenvalue decomposition (GEVD) [4] of pair $(\hat{\Sigma}_1, \hat{\Sigma}_X)$ such that

$$\hat{\Sigma}_1 \hat{\mathbf{u}}_1 = \lambda_1 \hat{\Sigma}_X \hat{\mathbf{u}}_1, \tag{9}$$

where $\lambda_1 = \hat{\rho}(\hat{\mathbf{u}}_1)$ is the largest generalized eigenvalue and $\hat{\mathbf{u}}_1$ is the associated eigenvector. As a consequence, from (6) and (7) $\hat{\mathbf{w}}_1$ can be expressed as $\hat{\mathbf{w}}_1 = R_X^T R_X \hat{\mathbf{u}}_1$ so that

$$\hat{\mathbf{w}}_1 = \hat{\Sigma}_X \hat{\mathbf{u}}_1, \tag{10}$$

with $\hat{\mathbf{u}}_1$ obtained from (9).

3. APPLICATION TO AN EFFICIENT SENSOR SELECTION

In the previous section, the theoretical analysis of the xDAWN algorithm allows to efficiently estimate the spatial filter and the spatial distribution related to the target evoked potentials. This section describes how to exploit this theoretical analysis to select efficiently and elegantly relevant sensors in a P300 speller BCI.

The algorithm to adaptively select relevant sensors is based on a recursive sensor elimination (backward elimination). Let us denote by $\mathcal{I}$ the set of M remaining sensors to be ranked at the j-th iteration of the backward elimination (i.e. $M = \text{card}(\mathcal{I})$). Each of the M remaining sensors is dropped alternatively and yields a performance score based on testing the subset $\mathcal{I}_{(-i)} = \mathcal{I} \setminus \{i\}$ made of the remaining $M - 1$ sensors ($\setminus$ is the minus set operator). Then, selecting the subset with the highest score $\rho_{\mathcal{I}_{(-i)}}$ means that the least relevant sensor is eliminated

$$\hat{i} = \arg\max_{i \in \mathcal{I}} \rho_{\mathcal{I}_{(-i)}}. \tag{11}$$

The rank of this sensor $r(\hat{i})$ is defined by $N_s - j$ where j is the iteration number at which the sensor has been dropped. This recursive procedure is pursued until all sensors have been eliminated, one by one. We thus obtain an ordering of all sensors according to their relevance to the task.

Classical methods used classification accuracy as performance score: for each iteration and each subset, a classifier is trained and the performance score is the classification accuracy achieved on a test database (i.e. using different data than those used to train the classifier to avoid over-fitting). The major drawback of this approach is its need for a large amount of data to train and to test the classifiers to avoid over-fitting. As a consequence, the ranking procedure of the sensors is computationally cumbersome and time consuming. To

overcome these drawbacks, we have proposed to use the estimated SSNR (5) as the performance score [2, 1] since this criterion has been proved to be more efficient to select relevant sensors.

In this study we propose to simplify the estimation of the estimated SSNR (5) by avoiding the GEVD computation for each sensor subset and at each computational step of the backward elimination which turns out to be highly time consuming. Indeed, from (10) the SSNR (3) can be expressed as

$$\rho(\hat{\mathbf{u}}_1) = \sigma_1^2 \hat{\mathbf{w}}_1^T \hat{\Sigma}_X^{-1} \hat{\mathbf{w}}_1. \tag{12}$$

As a consequence, to rank j-th sensor, one has to compute for all $i \in \mathcal{I}$ the SSNR $\rho_{\mathcal{I}_{(-i)}}$ given by

$$\rho_{\mathcal{I}_{(-i)}} = \sigma_1^2 \left(\hat{\mathbf{w}}_1^{\mathcal{I}_{(-i)}}\right)^T \left(\hat{\Sigma}_X^{\mathcal{I}_{(-i)}}\right)^{-1} \left(\hat{\mathbf{w}}_1^{\mathcal{I}_{(-i)}}\right), \tag{13}$$

vector $\hat{\mathbf{w}}_1^{\mathcal{I}_{(-i)}}$ (resp. $\hat{\Sigma}_X^{\mathcal{I}_{(-i)}}$) is defined from $\hat{\mathbf{w}}_1$ (resp. $\hat{\Sigma}_X$) by selecting components in $\mathcal{I}_{(-i)}$. Furthermore, since σ_1^2 is independent of spatial filter $\hat{\mathbf{u}}_1$ and remains invariant at each step of the backward elimination, it can be omitted to compute (13), which thus only requires the estimations of $\hat{\mathbf{w}}_1$ and $\hat{\Sigma}_X$. To avoid the computation of the inverse of matrix $\hat{\Sigma}_X^{\mathcal{I}_{(-i)}}$ for each tested subset of sensors, one can use the following trick, where for sake of simplicity $\hat{\Sigma}_X^{\mathcal{I}}$, $\hat{\Sigma}_X^{\mathcal{I}_{(-i)}}$, $\hat{\mathbf{w}}_1^{\mathcal{I}}$ and $\hat{\mathbf{w}}_1^{\mathcal{I}_{(-i)}}$ are denoted Σ, $\Sigma_{(-i)}$, $\mathbf{w}$ and $\mathbf{w}_{(-i)}$ respectively. Let us permute the i-th and last columns and rows of Σ (resp. the i-th and last rows of $\mathbf{w}$) and denote the new matrix by Σ' (resp. $\mathbf{w}'$). Furthermore, let us decompose Σ' and $\mathbf{w}'$ into block matrices such that

$$\Sigma' = \begin{bmatrix} \Sigma_{(-i)} & \mathbf{r}_i \\ \mathbf{r}_i^T & \gamma_i^2 \end{bmatrix} \qquad \mathbf{w}' = \begin{pmatrix} \mathbf{w}_{(-i)} \\ w_i \end{pmatrix}$$

with $\Sigma_{(-i)} \in \mathbb{R}^{(M-1)\times(M-1)}$, $\mathbf{r}_i \in \mathbb{R}^{M-1}$ and $\mathbf{w}_{(-i)} \in \mathbb{R}^{M-1}$. As a consequence, the highest theoretical SSNR (13) obtained when the i-th sensor has been dropped is given by

$$\rho_{\mathcal{I}_{(-i)}} = \sigma_1^2 \, \mathbf{w}_{(-i)}^T \Sigma_{(-i)}^{-1} \mathbf{w}_{(-i)}.$$

The matrix inversion lemma will enable us to avoid the computing of the inverse of $\Sigma_{(-i)}$ for each tested subset:

$$\Sigma'^{-1} = \begin{bmatrix} \Sigma_{(-i)}^{-1} + \dfrac{\Sigma_{(-i)}^{-1}\mathbf{r}_i\mathbf{r}_i^T\Sigma_{(-i)}^{-1}}{\gamma_i^2 - \mathbf{r}_i^T\Sigma_{(-i)}^{-1}\mathbf{r}_i} & -\dfrac{\Sigma_{(-i)}^{-1}\mathbf{r}_i}{\gamma_i^2 - \mathbf{r}_i^T\Sigma_{(-i)}^{-1}\mathbf{r}_i} \\ -\dfrac{\mathbf{r}_i^T\Sigma_{(-i)}^{-1}}{\gamma_i^2 - \mathbf{r}_i^T\Sigma_{(-i)}^{-1}\mathbf{r}_i} & \dfrac{1}{\gamma_i^2 - \mathbf{r}_i^T\Sigma_{(-i)}^{-1}\mathbf{r}_i} \end{bmatrix}.$$

Indeed, since

$$\Sigma_{(-i)}^{-1} = \left(\Sigma_{(-i)}^{-1} + \frac{\Sigma_{(-i)}^{-1}\mathbf{r}_i\mathbf{r}_i^T\Sigma_{(-i)}^{-1}}{\gamma_i^2 - \mathbf{r}_i^T\Sigma_{(-i)}^{-1}\mathbf{r}_i}\right) - \frac{\left(-\dfrac{\Sigma_{(-i)}^{-1}\mathbf{r}_i}{\gamma_i^2 - \mathbf{r}_i^T\Sigma_{(-i)}^{-1}\mathbf{r}_i}\right)\left(-\dfrac{\mathbf{r}_i^T\Sigma_{(-i)}^{-1}}{\gamma_i^2 - \mathbf{r}_i^T\Sigma_{(-i)}^{-1}\mathbf{r}_i}\right)}{\left(\dfrac{1}{\gamma_i^2 - \mathbf{r}_i^T\Sigma_{(-i)}^{-1}\mathbf{r}_i}\right)}$$

where each term in the latter expression are blocks of Σ'^{-1}, the inversion of Σ' allows to compute $\rho_{\mathcal{I}_{(-i)}}$ (13) for each of the possible M subsets composed of $(M-1)$ sensors as

$$\rho_{\mathcal{I}_{(-i)}} = \mathbf{w}_{(-i)}^T A \mathbf{w}_{(-i)} - \frac{\left(\mathbf{b}^T\mathbf{w}_{(-i)}\right)^2}{c}, \tag{14}$$

where

$$\Sigma'^{-1} = \begin{bmatrix} A & \mathbf{b} \\ \mathbf{b}^T & c \end{bmatrix}$$

with $A \in \mathbb{R}^{(M-1)\times(M-1)}$, $\mathbf{b} \in \mathbb{R}^{M-1}$ and $c \in \mathbb{R}$. Note that the columns and rows of $\Sigma_X'^{-1}$ are reordered to define A, $\mathbf{b}$ and c for each subsets $\mathcal{I}_{(-i)}$ composed of $M-1$ sensors ($i \in \mathcal{I}$).

The complete backward elimination procedure to rank the sensors is summarized in algorithm 1.

Algorithm 1 Efficient sensor selection algorithm.

```
Initialization of the full set of sensors I = {1, ..., N_s}
Estimation of Σ̂_X^I ← Σ̂_X and ŵ_1^I ← ŵ_1 (10)
for j = N_s to 2 do
  Compute inverse of Σ̂_X^I ⇒ (Σ̂_X^I)^{-1}
  for each i ∈ I do
    Compute ρ_{I_(-i)} from (14)
  end for
  Estimate the less significant sensor î as (11)
  Exclude this sensor: I ← I_(-î)
  Update Σ̂_X^I ← Σ̂_X^{I_(-î)} and ŵ_1^I ← ŵ_1^{I_(-î)}
  Rank removed sensor î: r(î) = N_s − j
end for
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4. RESULTS

4.1 Data acquisition, pre-processing and classification

Data were recorded from 20 healthy subjects [7]. Data from two separate sessions were considered: one dataset to select the most relevant sensors and to train the spatial filters and classifier, the second dataset to test the approach and evaluate the classification accuracy. In the training and test sessions, each subject had to write 50 and 60 predetermined symbols, respectively. Each row and column in the spelling matrix was randomly intensified for 100ms and the delay between two consecutive intensifications was 80ms, leading to an interstimulus interval of 180ms. The spelling of each symbol was based on 10 consecutive repetitions ($N_{epoch} = 10$).

EEG signals were initially sampled at 100Hz. Data preprocessing included a fourth order bandpass filter between 1Hz and 20Hz.

Among the proposed classifiers that are considered for BCIs, Bayesian linear discriminant analysis (BLDA) [5] is chosen since it has been shown to be efficient. The feature vector $\mathbf{p}$ here corresponds to the concatenation of all time-course samples in the enhanced signals. The discriminant vector $\mathbf{d}$ is estimated from the set of pairs $\{\mathbf{p}_j, t_j\}_{1\leq j\leq 12N_cN_{epoch}}$ obtained from the N_c symbols in the training database, where t_j is the class index associated with the corresponding feature vector.

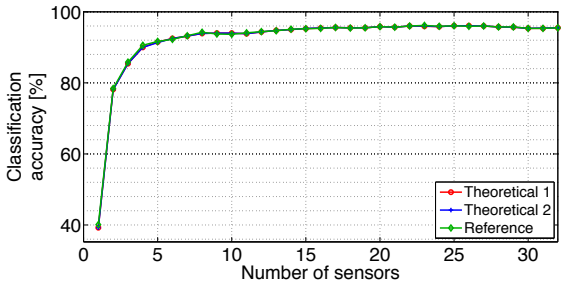

Figure 1: Average classification accuracy after 10 repetitions with respect to the number of selected sensors.

	Time (ms)	Speed up
Ref. Old	150037 ± 3189	1
Ref.	392 ± 9	383
Theo. 1	85 ± 1	1765
Theo. 2	153 ± 6	981

Table 1: Average computational time to rank sensor for one subject plus or minus the standard deviation.

4.2 Sensor selection

In this section, the influence of the computational method to select sensors is assessed. It is worth noting that comparison with other methods of sensor selection is not addressed in this study since it has already been proven that this procedure is at least as efficient as more classical ones [1].

Results of experiments are compared for three different performance score functions to rank the sensors. Let us refer to the classical method "Reference", which computes the GEVD at each step of the backward elimination and for each subset of sensors. This method is compared to the proposed method (labelled "Theoretical 1"). Finally, to quantify the influence of the inversion matrix lemma, a third method ("Theoretical 2"), which explicitly computes the inverse of $\hat{\Sigma}_X^{\mathcal{I}_{(-i)}}$, is implemented.

Figure 1 presents the classification accuracy versus the number of selected sensors. Obviously, the more sensors there are, the better the performance are. However, the number of sensors can be drastically decreased without a significant decreased of performance: e.g., with 10 repetitions, from 32 sensors to 8 sensors the decrease of performance is limited to 2% (96% to 94%) which remains acceptable for a BCI use. If only the best 75% of subjects are considered, from 32 sensors to 4 sensors the loss in classification accuracy is about 1%. Fortunately, this figure exhibits that the three methods provide quite similar rankings but not exactly the same: a t-test shows that at the 1% significance level there is no significative difference (p-value=.10 and .11 by comparing 'theorerical' methods to 'reference'). Fig. 2 details the classification accuracy for each subject. The results achieved with subjects 5, 12 and 17 are slightly different between the theoretical prediction and the practical selection. However these differences are not significant in average (the three curves are largely overlapped). This latter remark is confirmed by a pairwise two-tail t-test comparison which indicates that there is no significant differences between the three conditions ('theoretical 1', 'theoretical 2' and 'reference') since the null hypothesis is rejected. The slight differences can be explained by the fact that the theoretical SSNR (3) is derived from ideal covariance matrices Σ_1 and Σ_X which thus suppose perfect estimation of $\mathbf{w}_1$ and Σ_X while in practice we only have estimation of these parameters (10). Nevertheless, a t-test shows that this slight difference is not significative at the 1% significance level (p-value=0.43). Finally, this result is evidence that the proposed model (1) and the xDAWN algorithm used to estimate model parameters are successful since for almost all the subjects the sensors selected by the practical way are the same as those predicted by the theoretical analysis.

Table 1 reports the average computation time to complete the full backward elimination procedure to rank all the 32 sensors for one subject. In this table an additional method is added ("Reference Old"): it corresponds to the classical method which computes the two QR factorizations and the SVD at each step of the backward elimination. As one can see, the reformulation of the estimation of the SSNR by a GEVD instead of two QR and a SVD is definitively faster since it allows an impressive 383 speed up factor compared to previous implementations. Moreover, the theoretical analysis of the xDAWN algorithm addressed in this study decreases the computational time even more. Indeed, the theoretical method provides a speed up factor of 1765 compared to 'Ref. Old' (equivalent to a speed up factor of 4.6 compared to 'Ref') leading to a sensor ranking in less than 100ms on a standard PC (3GHz with Matlab R2009b). Moreover, the use of the inverse matrix lemma ("Theoretical 1") as explained in Section 3 allows further speed up the selection by a factor 1.8 compared to explicit computation of the inverse matrix ("Theoretical 2"). It is worth mentioning that with the new theoretical approach, only 85ms are necessary to rank the 32 sensors with an amount of about 20 minutes of data, while with the old implementation more than 2 minutes are required. This latter remark shows that it is now possible to monitor the influence of each sensor in real time which can be useful to provide a feedback to the user.

5. CONCLUSION AND PERSPECTIVES

In this study a theoretical analysis of the xDAWN algorithm has been addressed. The survey of the subjacent model in the P300 brain computer interface leads first to a more efficient implementation of the estimation of the spatial filters thanks to a generalized eigenvalue decomposition instead of two QR factorizations and a singular value decomposition. Moreover, this analysis provides an efficient implementation of the backward elimination to select sensors in the P300 speller BCI. This approach selects sensors such that the SSNR used as performance score function is the highest after the spatial filtering (xDAWN algorithm [9, 10]). The analysis of the model

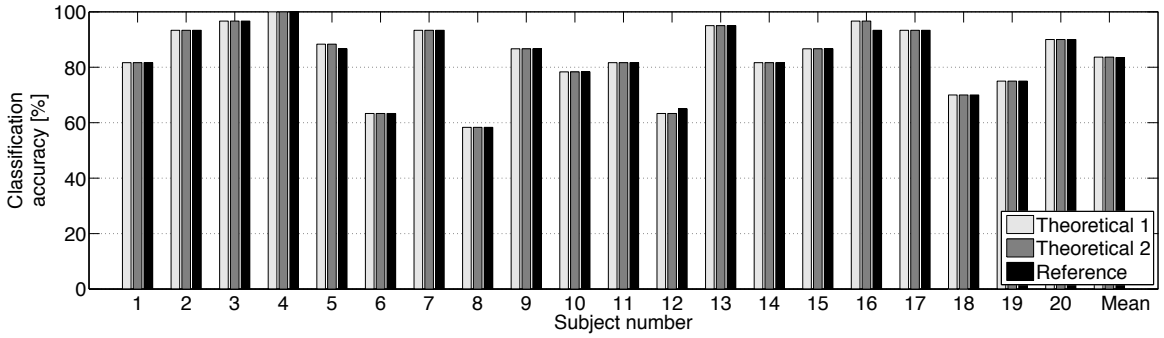

Figure 2: Comparison of the classification accuracy with 5 repetitions and 8 selected sensors.

in the P300 BCI leads to a theoretical prediction of this performance score function. This new theoretical implementation allows to speed up the backward elimination by a 4.6 factor compared to the classical direct estimation. This result is crucial in a real-life BCI since sensor selection is a very challenging aspect of BCI. Finally, this result also allows to dynamically manage the relevance of each sensor. For instance, to provide a real-time feedback of the importance of each sensor to track non-stationary process and could be used as an attentional marker.

6. ACKNOWLEDGMENTS

This work has been partially supported by the French National Research Agency (ANR) through the DEFIS program (project Co-Adapt ANR-09-EMER-002).

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